Individual welfare analysis for collective households*

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Abstract

We propose novel tools for the analysis of individual welfare on the basis of aggregate household demand behavior. The method assumes a collective model of household consumption with the public and private nature of goods specified by the empirical analyst. A main distinguishing feature of our approach is that it builds on a revealed preference characterization of the collective model that is intrinsically nonparametric. We show how to identify individual money metric welfare indices from observed household demand, along with the intrahousehold sharing rule and the individuals' willingness-to-pay for public consumption (i.e. Lindahl prices). The method is easy to use in practice and yields informative empirical results, which we demonstrate through both a simulation exercise and an empirical application to labor supply data drawn from the Panel Study of Income Dynamics.

Keywords: individual welfare; collective model; revealed preferences; sharing rule; money metric welfare index; identification; labor supply.

JEL codes: D11, D12, D13, C14

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1 Introduction

The analysis of individual welfare is at the core of the applied welfare literature given its relevance for a large variety of policy-relevant empirical questions. For example, when assessing inequality in a society, one of the basic objects of interest is the consumption level of individuals. If the within-household distribution of resources is highly unbalanced, inequality between individuals will be very different from inequality between aggregate households. In a similar spirit, it is individuals who have utilities and not households. This pleads for using measures of *individual* welfare when empirically evaluating the impact of policy reforms, such as tax reforms.

The empirical analysis of individual welfare raises two important challenges. Firstly, at the empirical level, the analyst usually only observes the aggregate household expenditures. The within-household sharing of resources is typically not observed.[1] Secondly, at the conceptual level, an important issue relates to the fact that households are intrinsically characterized by public consumption, which simultaneously benefits the different household members. The question remains how to evaluate this public consumption in the context of individual welfare analysis.

This paper presents a novel empirical method for the analysis of individual welfare that addresses both challenges. It is based on observed aggregate household consumption behavior, and it effectively accounts for intrahousehold public consumption in the evaluation of individual welfare.

Collective household consumption. We take as a starting point that the collective model of Apps and Rees (1988) and Chiappori (1988, 1992) provides a well-suited conceptual framework for dealing with these questions.[2] The attractive feature of this model is that it explicitly recognizes that households are not unitary decision making units, but consist of multiple decision makers with own rational preferences. Observed household consumption is regarded as the outcome of a within-household interaction process. The model (only) assumes that this process leads to Pareto-efficient intrahousehold allocations. Such a non-unitary approach to modeling households' consumption behavior is particularly relevant for the analysis of individual welfare, as it naturally allows us to account for the possibility of an unequal distribution of resources and welfare within

[1]In the past few years, more attention is given to the gathering of information on the consumption of individuals inside households (see, for example, Browning and Goertz (2012) and Cherchye, De Rock, and Vermeulen (2012)). Datasets with such information are still not widespread, though.

[2]The collective model has become the workhorse model in the family economics literature. It has been proven to be a viable alternative to the unitary model that is deficient when used in a context of multiperson decision making. See, for example, Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori et al. (2002), and Attanazio and Lechene (2014) for empirical evidence based on a parametric specification of household demand, and Cherchye and Vermeulen (2008) and Cherchye, De Rock, and Vermeulen (2009, 2011) for nonparametric evidence based on the revealed preference characterization of the collective consumption model.

households. See, for example, Chiappori and Meghir (2014) and Chiappori (2016) for extensive argumentation.

A main distinguishing feature of our method is that it builds on a revealed preference characterization of the collective model that is intrinsically nonparametric (in the tradition of Afriat (1967), Diewert (1973) and Varian (1982)). The method does not require an explicit parametric/functional specification of the intrahousehold decision process (e.g. individual preferences). This is particularly attractive from a conceptual point of view. From an empirical perspective, one potential disadvantage of this robust methodology is that the welfare-economic concepts will not be "point" identified but "set" identified (yielding lower and upper bounds on the individual welfare measures, as we explain in Sections 3 and 4). However, if the identified sets are tight (i.e. sharp upper and lower bounds), the practical relevance of this issue is low. Moreover, if the nonparametrically identified sets turn out to be wide, then this basically demonstrates that any more specific welfare-economic conclusion obtained from a parametric analysis is likely to depend heavily on the (nonverifiable) functional structure that is imposed.

Individual welfare analysis. We focus on a collective model with public and private consumption, in which the private and public nature of commodities is specified by the empirical analyst. This resembles the set-up of Chiappori and Ekeland (2009), who showed identifiability of all welfare-relevant aspects of this model under the exclusion restriction that, for each member, there exists at least one good that is not consumed by this member. Particularly, these authors showed that, if there are two exclusive goods and only public goods (i.e., no non-exclusive private goods), then the structural components of the model (including the individual utilities, individual prices and sharing rule) are completely identified. However, the strategy that we propose in the current paper in principle also admits non-exclusive private goods. Furthermore, we follow a nonparametric revealed preference approach, whereas Chiappori and Ekeland adopted a so-called differential approach. Our identification strategy yields robust nonparametric bounds instead of point estimates of the sharing rule and the MMWI.

We start from the revealed preference characterization of the collective model by Cherchye, De Rock, and Vermeulen (2011), and we develop a method that can provide the empirical tools for analyzing the individual welfare questions described above. First, we show how to identify the intrahousehold *sharing rule*, which defines the within-household distribution of resources.[3] Next, we build on this sharing rule identification to subse-

[3] The sharing rule takes a central position in empirical applications of collective consumption models. See, for example, Browning, Bourguignon, Chiappori, and Lechene (1994), Chiappori, Fortin, and Lacroix (2002), Blundell, Chiappori, and Meghir (2005), Lewbel and Pendakur (2008), Bourguignon, Browning, and Chiappori (2009), Couprie, Peluso, and Trannoy (2010), Lise and Seitz (2011), Bargain and Donni (2012), Cherchye, De Rock, and Vermeulen (2012), Browning, Chiappori, and Lewbel (2013) and Dunbar, Lewbel, and Pendakur (2013) for various applications of the collective consumption model that make use of the sharing rule concept.

quently identify the individuals' *money metric welfare indices*, which define the income that individuals need to be equally well off (in utility terms) as a single as in their current households. Chiappori and Meghir (2014) particularly advocated the use of these indices for individual welfare analysis based on the collective model in the presence of public goods.

As we will explain, both the sharing rule and the money metric welfare indices form special cases of the general concept of money metric utility, which is defined as the minimum amount of money at reference prices that an individual needs to attain a given welfare level. The difference lies in the reference prices that are used. While the sharing rule evaluates the expenditures on public goods at shadow (i.e. Lindahl) prices, money metric welfare indices evaluate these expenditures at market prices. Chiappori and Meghir (2014) argue that money metric welfare indices are especially well-suited for (intra individual) welfare comparisons, because they quantify welfare changes at constant prices for the given individuals. Variation in the sharing rule, by contrast, reflects not only changes in true welfare but also changes in the shadow prices, which are strongly context-dependent. For instance, these shadow prices will generally depend on the individual's current partner, and are subject to change when the individual becomes single or enters a new relationship. Therefore, the sharing rule is appropriate for intra-individual welfare comparisons only insofar as the individual's environment remains the same (e.g. to compute individual poverty rates). Money metric welfare indices, on the other hand, are robust to a change of environment and, in such a case, can capture the actual individual welfare changes more consistently (e.g. to compute the individual compensation that is needed to be equally well off after divorce as in the current marriage). This motivates our attempt to set identify not only the sharing rule but also money metric welfare indices.

We will demonstrate the practical usefulness of our identification tools by means of a simulation exercise, as well as through an empirical application to data drawn from the Panel Study of Income Dynamics (PSID). Our simulation exercise will illustrate the collective consumption mechanics underlying our identification method. Next, our empirical application is the first one that uses nonparametric revealed preference techniques to implement the collective money metric welfare concept advocated by Chiappori and Meghir (2014) for observational household consumption data. Through various exercises, we will show that our method allows for an informative empirical analysis. It has substantial empirical bite, despite its nonparametric orientation. For example, our results for the money metric welfare index enable us to quantify the households' economic gains through public consumption (i.e. scale economies), and to assess the effects of household income and relative wages on the intrahousehold (money metric) welfare distribution. In addition, we will show how to use our method to assess the prevalence of individual poverty, quantified in terms of both the sharing rule and money metric welfare indices.

Relation to previous work. In our empirical application, we will show that our identification method can be combined with nonparametric (e.g. Nadaraya–Watson) as well as parametric (e.g. QUAIDS) demand estimation, and that the method is straightforward to use in practice. In this sense, the current study complements the earlier work of Cherchye, De Rock, and Vermeulen (2011). The approach of these authors was designed for investigating a finite set of observed household consumption choices. The current study deviates from this earlier work by drawing consumption bundles from a demand function. This not only reduces the level of noise in the data. It also significantly increases the number of consumption bundles that serve as input for the revealed preference analysis, so substantially improving the informativeness of the identification results (i.e. sharp bounds on the sharing rule and the money metric welfare indices). In addition, it is worth noting that the approach of Cherchye, De Rock, and Vermeulen (2011) requires solving mixed integer linear programming (MILP) problems, while our novel method relies on enumeration techniques and linear programming only. It is well established that enumeration and linear programming problems are computationally easier to solve than MILP problems.

At the methodological level, this paper also complements recent work of Cherchye, De Rock, Lewbel, and Vermeulen (2015). Two main differences are that (1) this earlier paper focused on a general collective consumption model in which the (public or private) nature of the goods is left unspecified, and (2) the proposed method allows for sharing rule recovery but not for the identification of money metric welfare indices.[4] This last difference directly motivates the relevance of our contribution in the current paper because, as indicated above, money metric welfare indices have been advocated as particularly useful tools for individual welfare analyses in a collective consumption context. Moreover, in empirical applications it is often possible to formulate reasonable assumptions regarding the nature of goods prior to the actual empirical analysis. For instance, this applies to the labor supply setting that we will use as our leading example throughout this paper.

Outline. The rest of this paper unfolds as follows. In the next section, we define our concept of rational household behavior in terms of the collective consumption model. Section 3 presents our method for sharing rule (set) identification, while Section 4 shows how to subsequently identify individuals' money metric welfare indices. Section 5 presents our simulation analysis, and Section 6 our application to PSID data. Section 7 concludes.

[4]Technically, as we will explain below, the identification of money metric welfare indices requires the recovery of individuals' shadow/Lindahl prices of publicly consumed quantities. This recovery is not possible by using Cherchye, De Rock, Lewbel, and Vermeulen (2015)'s method for sharing rule identification when the (public or private) nature of goods is unknown. As a direct implication, the identification strategy that we develop in the current paper, which will allow us to recover individual-specific shadow prices, is substantively different from the one proposed by these authors.

2 Collective rationality

We assume a standard setting in which households consist of two decision makers, member 1 and member 2. The empirical analyst observes a set N of household decision situations, which are characterized by aggregate consumption quantities, associated prices and incomes. In our following simulation and empirical application, the set N will be drawn from the observed household demand function, which sets out the households' consumption quantities as a function of household level prices and incomes. Household consumption will be partly public and partly private. The public and private nature of each good is specified prior to the empirical analysis.

To simplify the exposition, our following analysis will assume decision situations with the number of commodities limited to three: good 1 is private and assignable to household member 1, good 2 is private and assignable to household member 2, and good 3 is publicly consumed. As a specific example, we use the non-unitary labor supply setting with goods 1 and 2 the male's and female's leisure, and good 3 the household consumption of a Hicksian aggregate. The fact that we model the Hicksian aggregate consumption as public consumption allows it to be interpreted as purely public consumption or, alternatively, as private consumption associated with (positive) externalities.

This three-goods setting was studied by Chiappori (1988, 1992) in his original papers, and will also be considered in our following simulation and empirical application. At this point, we note that, in principle, it is possible to extend our following reasoning to settings with more commodities. In fact, it may well be that some private goods are not assignable to individual household members. However, in general we need that at least one good is assignable to each member.

Formally, let l_E^1 and l_E^2 denote the time spent on leisure by members 1 and 2 in household decision situation $E \in N$. Further, Q_E is the amount of the public good in situation E. Finally, let w_E^1 and w_E^2 represent the individuals' wages (i.e. prices of leisure) and y_E the household's total expenditures on leisure and consumption, i.e. $y_E = w_E^1 l_E^1 + w_E^2 l_E^2 + Q_E$. Taken together, this defines the household data set

$$S = \{(w_E^1, w_E^2, 1); (l_E^1, l_E^2, Q_E)\}_{E \in N}.$$

We say that the data set S is *collectively rational* if each decision situation $E \in N$ can be represented as Pareto efficient, which means that the household maximizes a weighted sum of individual utility functions subject to a budget constraint.[5]

Definition 1 (Collective rationality). *The household data set S is collectively rational if there exist individual utility functions U^1 and U^2 and bargaining weights μ_E^1 and*

[5]Throughout, we assume that the individual utility functions U^1 and U^2 are differentiable, strictly monotone and strictly concave.

μ_E^2 *such that, for all decision situations* $E \in N$,

$$(l_E^1, l_E^2, Q_E) = \arg\max_{l^1,l^2,Q} \mu_E^1 U^1(l^1, Q) + \mu_E^2 U^2(l^2, Q)$$
$$s.t.$$
$$w_E^1 l^1 + w_E^2 l^2 + Q \leq w_E^1 l_E^1 + w_E^2 l_E^2 + Q_E.$$

The Pareto weights μ_E^1 and μ_E^2 in the objective function represent the relative bargaining power of the household members 1 and 2. We remark that these bargaining weights may vary depending on the decision situation E. Obviously, identifying these Pareto weights can give insight into the intrahousehold distribution of bargaining power. However, the value of these weights will depend on the cardinalization of the utility functions U^1 and U^2.

An intrinsic feature of the collective model is the so-called sharing rule, which governs the within-household distribution of resources. This sharing rule is often interpreted as an alternative indicator of the relative bargaining power of individual household members. Unlike the Pareto weights μ_E^1 and μ_E^2, an attractive feature of the sharing rule is that it is expressed in monetary terms.

In what follows, we will say that the utility functions U^1 and U^2 *collectively rationalize* the household data set S if they satisfy the associated restrictions in Definition 1. For such utility functions, we define θ_E^i $(i =, 2)$ as individual i's *shadow price* for the public consumption Q_E, which intuitively corresponds to the individual's willingness-to-pay for this consumption.

Definition 2 (Shadow price for public consumption). *For individual utility functions U^1 and U^2 that collectively rationalize the household data set S, the shadow prices* θ_E^1 and θ_E^2 *represent the marginal rates of substitution between leisure and public consumption of individuals* 1 *and* 2, *i.e.* $\theta_E^i = \dfrac{w_E^i \, \partial U^i/\partial Q_E}{\partial U^i/\partial l_E^i}$ *for* $i = 1, 2$.

Pareto efficiency implies that the shadow prices θ_E^1 and θ_E^2 can be interpreted as Lindahl prices and therefore requires these prices to sum to the price of the household's public consumption (i.e. $\theta_E^1 + \theta_E^2 = 1$, for 1 equal to the price of the Hicksian public good). Finally, for our discussion below, it will be useful to directly define collective rationality in terms of the sharing rule. To this end, we formally specify individual i's $(i = 1, 2)$ expenditure/consumption share as

$$\eta_E^i = w_E^i l_E^i + \theta_E^i Q_E,$$

which comprises the individual's leisure component $w_E^i l_E^i$ and a share θ_E^i of the public consumption Q_E. Then, the Second Fundamental Theorem of Welfare Economics obtains the following equivalent definition of collective rationality.

Definition 3 (Collective rationality: sharing rule representation). *The household data set S is collectively rational if there exist individual utility functions U^1 and U^2, expenditure shares η_E^1 and η_E^2 and shadow prices θ_E^1 and θ_E^2 such that, for all decision situations $E \in N$,*

$$
\begin{gathered}
(l_E^i, Q_E) = \arg\max U^i(l^i, Q) \\
s.t. \\
w_E^i l^i + \theta_E^i Q \leq \eta_E^i,
\end{gathered}
$$

with $\theta_E^1 + \theta_E^2 = 1$.

This definition provides a "decentralized" expression of collective rationality. It shows that, for a given sharing rule (defining η_E^1 and η_E^2), collective rationality imposes individually rational (i.e. utility maximizing) behavior of each household member separately.

3 Sharing rule recovery

In this section, we start from Definition 3 of collective rationality to address identification of the individual shares η_E^1 and η_E^2 associated with an observed decision situation E. Basically, the method obtains recovery of these individual shares under the maintained assumption that the observed household consumption behavior satisfies collective rationality. In particular, we will show that we can define upper and lower bounds on the expenditure shares by starting from a nonparametric revealed preference characterization of the collective consumption model. The fact that we define bounds effectively obtains "set" identification (in contrast to "point" identification) of the household's sharing rule.

More specifically, our method recovers bounds on the shadow prices θ_E^i, which directly implies bounds on η_E^i $(= w_E^i l_E^i + \theta_E^i Q_E)$. The method builds on the individual rationality requirement in Definition 3. The revealed preference characterization of this rationality condition will define inequality restrictions for observed demand behavior, which in turn will lead to set identification of θ_E^i. By exploiting the revealed preference implications of collective rationality, we can shrink the region of member i's shadow prices in household E from the trivial "uninformative" interval $[0,1]$ to an "informative" interval $\Theta_E^i = [\theta_E^{i,lb}, \theta_E^{i,ub}]$ with $0 \leq \theta_E^{i,lb} \leq \theta_E^{i,ub} \leq 1$.

As we will explain, our bounds will not necessarily represent the tightest bounds that can be obtained by exploiting all empirical restrictions implied by collective rationality. To formalize this point, we let the set Φ_E^i represent these tightest bounds.

Definition 4 (The set Φ_E^i). *For a given data set S, we have $\Phi_E^i = [\tilde{\theta}_E^{i,lb}, \tilde{\theta}_E^{i,ub}]$ $(i = 1, 2)$ where*

- $\tilde{\theta}_E^{i,lb}$ *is the lowest shadow price* θ_E^i *associated with individual utility functions* U^1 *and* U^2 *that collectively rationalize the set* S*, and*

- $\tilde{\theta}_E^{i,ub}$ *is the highest shadow price* θ_E^i *associated with individual utility functions* U^1 *and* U^2 *that collectively rationalize the set* S*.*

In words, $\tilde{\theta}_E^{i,lb}$ is the minimum marginal rate of substitution between leisure and public consumption for rationalizing utility functions U^1 and U^2, and $\tilde{\theta}_E^{i,ub}$ the maximum marginal rate of substitution. In our following reasoning, we will construct the set Θ_E^i such that $\Phi_E^i \subseteq \Theta_E^i$, i.e. Θ_E^i provides an empirical outer bound approximation of the theoretically tightest set Φ_E^i. We will return to sharpness of our empirical bounds (defining Θ_E^i) at the end of this section. Importantly, even though our bounds are not necessarily the tightest possible bounds in theory, they will have substantial empirical bite, as we will show in our simulation analysis in Section 5 and our application in Section 6. In addition, they will be very easy to compute in practice.

Throughout, we will assume to have an arbitrarily large set N of household consumption bundles with associated prices. For example, in our simulation exercise and empirical application in Sections 5 and 6, these bundles will be drawn from a household demand function g that maps realizations of w^1, w^2 and y on $(l^1, l^2, Q) = g(w^1, w^2, y)$. At this point, we remark that, in principle, we could also have expressed our following argument directly in terms of the continuous demand function g rather than in terms of the discrete set N (along the lines of Cherchye, De Rock, Lewbel, and Vermeulen (2015)). However, using the discrete set N substantially facilitates our exposition. Moreover, as we will explain below, it implies a very simple enumeration method to obtain Θ_E^i. Finally, it shows that our proposed method is also directly applicable to settings with discrete sets of household observations (as originally considered by Cherchye, De Rock, and Vermeulen (2011)).

Individual (ir)rationality for given shadow prices. The basic idea is to exploit that violations of individual rationality imply violations of collective rationality. That is, the shadow price θ_E^i is not sustainable when it leads to a violation of individual rationality for individual i in situation E. In that case, we conclude that $\theta_E^i \notin \Phi_E^i$. In what follows, we characterize these unsustainable θ_E^i in revealed preference terms, and we will use this characterization to define the upper bound $\theta_E^{i,ub}$. Correspondingly, we can define the lower bound $\theta_E^{i,lb} = 1 - \theta_E^{j,ub}$ $(j \neq i)$, by using the adding up condition $\theta_E^1 + \theta_E^2 = 1$ for Lindahl prices.

More precisely, let us assume some given shadow prices θ_E^1 and θ_E^2. Then, Definition 3 simultaneously imposes individual rationality on both household members: there must exist utility functions U^1 and U^2 such that (l_E^1, Q_E) maximizes U^1 and (l_E^2, Q_E) maximizes U^2 subject to individual budget constraints. Failure to find U^i for at least one of the household members results in a rejection of collective rationality for the specified θ_E^i, which means $\theta_E^i \notin \Phi_E^i$. We can rephrase this in revealed preference terms, by using that a necessary condition for individual rationality is that the data are consistent with the Weak Axiom of Revealed Preference (WARP). For our setting, WARP consistency of individual i requires that, for all $n, n' \in N$ such that $(l_n^i, Q_n) \neq (l_{n'}^i, Q_{n'})$, there must exist shadow prices θ_n^i and $\theta_{n'}^i$ that meet

$$w_n^i l_n^i + \theta_n^i Q_n \geq w_n^i l_{n'}^i + \theta_n^i Q_{n'} \Rightarrow w_{n'}^i l_n^i + \theta_{n'}^i Q_n > w_{n'}^i l_{n'}^i + \theta_{n'}^i Q_{n'}. \tag{1}$$

Suppose that the household data set S is collectively rational, meaning that the set Φ_E^i is non-empty. For the given set Φ_E^i, a sufficient condition for θ_E^i to be inconsistent with condition (1) is that, for some $n \in N$,

$$w_E^i(l_n^i - l_E^i) + \theta_E^i(Q_n - Q_E) < 0 \text{ and} \tag{2}$$

$$\forall \theta_n^i \in [0,1] : w_n^i(l_E^i - l_n^i) + \theta_n^i(Q_E - Q_n) < 0. \tag{3}$$

If conditions (2) and (3) hold simultaneously, then we conclude that, for any specification of θ_n^i, the WARP requirement (1) is violated for the given θ_E^i. As a direct implication, we have that $\theta_E^i \notin \Phi_E^i$.[6]

Finally, by replacing the second term on the left hand side of inequality (3) by its maximum and minimum potential values (i.e. zero and $(Q_E - Q_n)$), we obtain the two inequalities

$$w_n^i(l_E^i - l_n^i) \quad < \quad 0 \text{ and} \tag{4}$$

$$w_n^i(l_E^i - l_n^i) + (Q_E - Q_n) \quad < \quad 0, \tag{5}$$

which hold simultaneously if and only if (3) holds. This reformulation will be useful in practical applications, as it avoids having to implement the universal quantifier in (3) (for the unknown θ_n^i).

Defining $\Theta_E^i = [\theta_E^{i,lb}, \theta_E^{i,ub}]$. Using the above, we can define a straightforward enumeration method to compute the upper bound $\theta_E^{i,ub}$ and lower bound $\theta_E^{i,lb}$. Specifically, we solve

[6] We remark that we work with a strict inequality in condition (2). We do so because it facilitates the empirical implementation of our method. However, it also implies that, strictly speaking, we are not exhausting all empirical implications associated with the WARP condition in (1).

$$\theta_E^{i,ub} = \min\left(1\ ;\ \min_{n\in\Omega_E^i, Q_n<Q_E} \frac{w_E^i(l_n^i - l_E^i)}{(Q_E - Q_n)}\right), \tag{6}$$

$$\text{with } \Omega_E^i = \left\{ n \in N| \begin{array}{c} w_n^i(l_E^i - l_n^i) < 0, \\ w_n^i(l_E^i - l_n^i) + (Q_E - Q_n) < 0 \end{array} \right\}. \tag{7}$$

The reasoning (which underlies our proof of Proposition 1) goes as follows. We first identify the set Ω_E^i. Following our above argument (leading up to (4) and (5)), in (7) we guarantee that each observation $n \in \Omega_E^i$ satisfies (3). Thus, WARP-consistent values of θ_E^i cannot satisfy $w_E^i(l_n^i - l_E^i) + \theta_E^i(Q_n - Q_E) < 0$ for $n \in \Omega_E^i$ (see (2)). Because $l_E^i < l_n^i$ for any $n \in \Omega_E^i$ (see (4)), this last inequality requirement implies an informative upper bound on θ_E^i only if $Q_E > Q_n$. Then, focusing on $n \in \Omega_E^i$ with $Q_E > Q_n$, in (6) we define a "most informative" (i.e. lowest) upper bound as $\theta_E^{i,ub} = \min_{n\in\Omega_E^i, Q_n<Q_E} \frac{w_E^i(l_n^i - l_E^i)}{(Q_E - Q_n)}$. Finally, we correct our procedure to deal with situations where the set Ω_E^i is empty (e.g. because $l_E^i > l_n^i$ for all $n \in N$) or $\theta_E^{i,ub} > 1$. In that case, we conclude that our procedure does not identify an informative upper bound and, therefore, we set the uninformative bound $\theta_E^{i,ub} = 1$ in (6).

By running a similar enumeration method, we can compute $\theta_E^{j,ub}$ for member $j \neq i$, which allows us to define the lower bound $\theta_E^{i,lb} = 1 - \theta_E^{j,ub}$. In turn, this obtains $\Theta_E^i = [\theta_E^{i,lb}, \theta_E^{i,ub}]$. As a last remark, we have that, by construction, the observation E is not compatible with the model of collective rationality (i.e. violates the WARP requirement in (1)) if $\theta_E^{i,lb} > \theta_E^{i,ub}$. In that case, we can set $\Theta_E^i = \varnothing$.

We can now state our first main result.[7]

Proposition 1 *We have that* $\Phi_E^i \subseteq \Theta_E^i = [\theta_E^{i,lb}, \theta_E^{i,ub}]$.

Sharpness. In general, we have $\Phi_E^i \subseteq \Theta_E^i$, which means that Θ_E^i need not necessarily exploit all the theoretical implications of collective rationality. One reason is that the computation of θ_E^{lb} and θ_E^{ub} is based on WARP, which only captures necessary implications of individually rational behavior. As shown by Houthakker (1950), utility maximization generally implies that the Strong Axiom of Revealed Preference (SARP) holds. SARP extends WARP by also exploiting transitivity of preferences.[8] Our focus on WARP instead of SARP follows Cherchye, De Rock, Lewbel, and Vermeulen (2015). As argued by these authors, when exploiting SARP, it would be much more difficult, if not completely

[7] See Appendix A for the proofs of our Propositions 1, 2 and 3.

[8] The method to define bounds on individuals' money metric welfare indices that we introduce in Section 4 will also be based on WARP instead of SARP. In particular, our proofs of Propositions 2 and 3 will mainly use WARP-based revealed preference arguments (see Appendix A). See also Smeulders, Cherchye, De Rock, Spieksma, and Talla Nobibon (2015) who study the relationship between WARP and SARP for the labor supply setting on which we focus here.

intractable, to fully operationalize transitivity in an empirical application. Moreover, in Sections 5 and 6 we will show that the WARP-based bounds produced by our methodology are informatively tight.

4 Money metric welfare indices

For a given decision situation E, the previous section proposed a method that set identifies individual income shares η_E^1 and η_E^2 by recovering sets of shadow prices Θ_E^1 and Θ_E^2 that are consistent with our assumption of collective rationality. We next show that we can use these bounds on the shadow prices θ_E^i for each individual i to define informative upper and lower bounds on individuals' money metric welfare indices ($MMWI$s). As indicated in the Introduction, Chiappori and Meghir (2014) advocated the use of these $MMWI$s for individual welfare analysis based on the collective model.

Theoretical concept. In words, an individual's $MMWI$ computes the minimum income needed by a household member to achieve the intrahousehold utility level (expressed in terms of material consumption) when the individual comes to live alone. Formally, for individual i in the observed decision situation E (with intrahousehold allocation (l_E^1, l_E^2, Q_E)), we have

$$MMWI_E^i = \min_{l^i,Q}\{w_E^i l^i + Q |\ (l^i, Q) \in B^i(l_E^i, Q_E)\},$$

where $B^i(l_E^i, Q_E)$ represents the better-than-set associated with the given bundle (l_E^i, Q_E), i.e.

$$B^i(l_E^i, Q_E) = \{(l^i, Q)|\ U^i(l^i, Q) \geq U^i(l_E^i, Q_E)\}.$$

Basically, this $MMWI_E^i$ calculates the minimal expenditures over the bundles that are at least as good as the bundle (l_E^i, Q_E). In doing so, it accounts for the fact that, when becoming single, the individual will have to bear the full cost (and no longer the individual's shadow cost θ_E^i) for the publicly consumed good. To recall, the single's price for the (Hicksian) public good equals 1.

We remark that obviously $(l_E^i, Q_E) \in B^i(l_E^i, Q_E)$ and, by construction, the individual cannot face a price decrease of the Hicksian good when becoming single (because $\theta_E^i \leq 1$). Using this, one can easily show that $(l^i, Q) \in B^i(l_E^i, Q_E)$ with $Q > Q_E$ implies $w_E^i l^i + Q \geq w_E^i l_E^i + Q_E$. Therefore, to compute $MMWI_E^i$ it suffices to only minimize expenditure over all bundles $(l^i, Q) \in B^i(l_E^i, Q_E)$ with $Q \leq Q_E$, since bundles $(l^i, Q) \in B^i(l_E^i, Q_E)$ with $Q > Q_E$ are always more expensive than (l_E^i, Q_E). We use this argument in Propositions 2 and 3 below.

Upper bound on $MMWI_E^i$. To define an informative upper bound on $MMWI_E^i$ we make use of the theoretical restrictions of collective rationality. The construction of the upper bound $m_E^{i,ub}$ requires an empirical inner bound approximation $IB^i(l_E^i, Q_E)$ of the unknown better-than-set $B^i(l_E^i, Q_E)$. This use of an inner bound set $IB^i(l_E^i, Q_E)$ to define an upper bound for $MMWI_E^i$ parallels Varian (1982)'s procedure to compute bounds on money metric utilities in a unitary household consumption context. The same remark applies to the outer bound approximation $OB^i(l_E^i, Q_E)$ that we will use to define a lower bound for $MMWI_E^i$. Basically, our method provides a collective version of Varian's original method.

Next, our construction of the empirical inner bound $IB^i(l_E^i, Q_E)$ makes use of the upper bound $\theta_n^{i,ub}$ on the shadow prices. More specifically, we define

$$IB^i(l_E^i, Q_E) = \{(l_n^i, Q_n) | n \in N, Q_n \leq Q_E \text{ and } w_n^i(l_n^i - l_E^i) + \theta_n^{i,ub}(Q_n - Q_E) \geq 0\}.$$

In words, because we use the upper bound on the shadow prices that sustain collective rationality, we can always conclude that (l_n^i, Q_n) is revealed preferred over (l_E^i, Q_E) if $w_n^i(l_n^i - l_E^i) + \theta_n^{i,ub}(Q_n - Q_E) \geq 0$. By construction, the last inequality will also be satisfied for the true (but unobserved) shadow price (which is situated below $\theta_n^{i,ub}$). Thus, we obtain $(l_n^i, Q_n) \in B^i(l_E^i, Q_E)$ as soon as $w_n^i(l_n^i - l_E^i) + \theta_n^{i,ub}(Q_n - Q_E) \geq 0$. This is formally stated in the next proposition.

Proposition 2 *We have that $IB^i(l_E^i, Q_E) \subseteq B^i(l_E^i, Q_E)$.*

Based on Proposition 2, we can then define the upper bound

$$m_E^{i,ub} = \min_n \{w_E^i l_n + Q_n | \ (l_n^i, Q_n) \in IB^i(l_E^i, Q_E)\}.$$

Note that a simple enumeration procedure can be used to compute $IB^i(l_E^i, Q_E)$ and, correspondingly, $m_E^{i,ub}$. This is attractive from an operational point of view.

Lower bound on $MMWI_E^i$. The construction of the empirical outer bound $OB^i(l_E^i, Q_E)$ is slightly more complicated and makes use of both the lower bound $\theta_n^{i,lb}$ and the upper bound $\theta_n^{i,ub}$ on the unknown shadow prices for public consumption. More specifically, we define

$$\begin{aligned} OB^i(l_E^i, Q_E) = \{(l^i, Q) | w_n^i(l_n^i - l^i) + \theta_n^{i,lb}(Q_n - Q) \leq 0 \text{ for all } n \in N \text{ for which} \\ w_E^i(l_E^i - l_n^i) + \theta_E^{i,lb}(Q_E - Q_n) \geq 0 \text{ if } Q_n < Q_E \text{ or} \\ w_E^i(l_E^i - l_n^i) + \theta_E^{i,ub}(Q_E - Q_n) \geq 0 \text{ if } Q_n \geq Q_E\}. \end{aligned}$$

Similar to before, the last two inequality constraints ensure that, for a given n, we can always conclude that (l_E^i, Q_E) is revealed preferred over (l_n^i, Q_n), while the first inequality constraint implies that we can never conclude that (l_n^i, Q_n) is strictly revealed preferred over (l^i, Q). Together this implies that we cannot conclude that (l_E^i, Q_E) is strictly revealed preferred over (l^i, Q). As a direct consequence, we cannot exclude (l^i, Q) from $B^i(l_E^i, Q_E)$. This yields the following result.

Proposition 3 *We have that* $\{(l^i, Q) \in B^i(l_E^i, Q_E) | Q \leq Q_E\} \subseteq OB^i(l_E^i, Q_E)$.

From this proposition, we can define the lower bound

$$m_E^{i,lb} = \min_{l^i, Q} \{w_E^i l^i + Q | \ (l^i, Q) \in OB^i(l_E^i, Q_E)\}.$$

Again it is straightforward to compute this lower bound. In this case, it suffices to solve a simple linear program with a condition $w_n^i(l_n^i - l^i) + \theta_n^{i,lb}(Q_n - Q) \leq 0$ for each $n \in N$ that satisfies one of the last inequality restrictions in our definition of $OB^i(l_E^i, Q_E)$.

Money metric utility. The sharing rule and $MMWI$ concepts are two particular instances of money metric utility. Formally, money metric utility is defined as the minimum expenditure $e^i(p; u)$ required for individual i to attain some given utility u at reference prices p. By this definition, we can express individual i's income share η_E^i and money metric welfare $MMWI_E^i$ as

$$\begin{aligned} \eta_E^i &= e^i(w_E^i, \theta_E^i; U^i(l_E^i, Q_E)), \text{ and} \\ MMWI_E^i &= e^i(w_E^i, 1; U^i(l_E^i, Q_E)). \end{aligned}$$

In words, η_E^i equals the minimum expenditure needed for individual i to purchase a consumption bundle that is at least as good as (l_E^i, Q_E), with public consumption valued at the shadow prices θ_E^i. Next, $MMWI_E^i$ also equals the minimum expenditure needed for individual i to purchase a consumption bundle that is at least as good as (l_E^i, Q_E), but public consumption is now valued at market prices (in casu 1).

Chiappori and Meghir (2014) showed that, when all commodities are private, there is a one-to-one correspondence between individual income shares and utility (for a given vector of commodity prices). In that case, the sharing rule provides a satisfactory money metric of individual welfare. However, in more general settings with public goods, shadow/Lindahl prices depend on a complicated interaction of budgets, bargaining weights and *all* preferences in the household. Keeping Lindahl prices fixed requires conditioning on all these factors, which rules out welfare comparisons for the same individual in different situations. By contrast, the $MMWI$ concept keeps reference prices constant and equal to the market price. Intra personal welfare comparisons no longer depend on the (variable) Lindahl prices. There is a one-to-one correspondence between an individual's $MMWI$ and his/her utility, irrespective of specific household characteristics.

This makes the $MMWI$ concept better suited for intra individual welfare comparisons with public goods. See also and Decancq, Fleurbaey, and Schokkaert (2015) and Capéau (2017) for related discussions.

5 Simulation analysis

To investigate the empirical performance of our revealed preference method, we begin by conducting a simulation analysis. This simulation exercise serves to illustrate the intrahousehold collective consumption mechanics that underlie our identification method, which will also facilitate the interpretation of our empirical results in Section 6. To do so, we will assume a fairly unsophisticated parametric specification of the individual preferences and the bargaining process. We will consider the tightness of the bounds that our method recovers for the within-household consumption sharing pattern (i.e. the individuals' shadow/Lindahl prices and associated expenditure shares) and for the individuals' money metric welfare indices. Attractively, we will conclude that, even for our non sophisticated parametric setting, our method generates bounds that are close to the true individual shares η^i_E, Lindahl prices θ^i_E and welfare indices $MMWI^i_E$.

Set-up. Following our theoretical exposition, we assume a setting with three commodities, i.e. (private and assignable) leisure of the two spouses and remaining (public) Hicksian consumption. The individuals' utility functions take the Cobb–Douglas form

$$\begin{aligned} U^1(l^1, Q) &= \alpha \ln l^1 + (1-\alpha) \ln Q, \\ U^2(l^2, Q) &= \beta \ln l^2 + (1-\beta) \ln Q. \end{aligned}$$

In this simple specification, the parameters $0 \leq \alpha, \beta \leq 1$ define the individuals' preferences over private and public consumption. Generally, higher values for α and β reflect stronger individual preferences for leisure. In what follows, we will use $\alpha = 1/2$ and $\beta = 1/4$, meaning that household member 2 has stronger preferences for public consumption than household member 1.

As explained before, collective rationality means that the household consumption bundle (l^1_E, l^2_E, Q_E) maximizes a weighted sum of the individual utility functions (i.e. $\mu^1_E U^1 + \mu^2_E U^2$) subject to the household budget constraint. Here, we will assume that individual bargaining weights depend on the individual wages (as prices of leisure), by using $\mu^1_E = 1$ and

$$\mu^2_E = \frac{3}{2}\frac{w^2_E}{w^1_E}.$$

The intuition is straightforward: the higher the individual's wage, the stronger his or her bargaining position. This positive relationship between an individual's relative wage and his/her bargaining weight has broad empirical support in the literature on collective consumption models. See, for example, Browning, Chiappori, and Weiss (2014) for a recent review.

In what follows, we will specifically focus on a household decision situation E with aggregate income $y_E = 19.5$ and wages $w_E^1 = 0.25$ and $w_E^2 = 0.5$. This corresponds to a bargaining weight $\mu_E^2 = 3$, indicating that household member 2 has a stronger bargaining position than member 1.

Intrahousehold sharing. For the given parametric specification and budget conditions, we can directly define the individuals' "true" Lindahl prices θ_E^i and expenditure shares η_E^i. In our case, we obtain

$$\theta_E^1 = 0.182, \theta_E^2 = 0.818, \eta_E^1 = 4.875 \text{ and } \eta_E^2 = 14.625,$$

corresponding to a household consumption bundle $(l_E^1, l_E^2, Q_E) = (9.750, 7.315, 13.406)$. The fact that member 1 contributes less to the public good than member 2 is not surprising, given that this individual has a weaker preference for public consumption. Next, the higher expenditure share of individual 2 reflects his/her better bargaining position.

Let us then investigate how well the bounds obtained through our empirical method (outlined in Section 2) approximate the above theoretical values for θ_E^i and η_E^i. In particular, we focus on tightness of the sets $\Theta_E^i = [\theta_E^{i,lb}, \theta_E^{i,ub}]$. As explained in Section 2, tight bounds for θ_E^i directly translate into similarly tight bounds for η_E^i.

To apply our identification procedure, we simulate a large set N of bundles (l^1, l^2, Q) that are collectively rational (for the given utilities and bargaining weights) under alternative regimes of the wages w^1, w^2 and income y.[9] This resulted in the following bounds

$$\Theta_E^1 = [0.171, 0.200] \text{ and } \Theta_E^2 = [0.800, 0.830].$$

As a first observation, we note that these sets effectively contain the true values $\theta_E^1 = 0.182$ and $\theta_E^2 = 0.818$, which empirically confirms our theoretical result in Proposition 1. Next, and more interestingly, we observe that the bounds are very tight, which obtains fairly precise set identification. This shows that our recovery method can allow for a significantly informative analysis of the within-household distribution of individuals' Lindahl prices (and correspondingly resources). Our empirical application in the next section will show that this attractive feature also holds in real-life settings.

[9] Specifically, our following identification results for individuals' income shares, Lindahl prices and money metric welfare indices are based on $|N| = 8000$. Details on our procedure to draw wages w^1, w^2 and incomes y are available upon request. As explained in Sections 3 and 4, our identification methods require simple (enumeration) procedures, which makes it easy to consider large $|N|$.

Money metric welfare. We next turn to identification of the individuals' money metric welfare indices $MMWI_E^i$. As a preliminary step, we again compute the "true" values of these indices for our parametric specification, the given prices and the household income. In this application, these indices capture the income that individuals would need as singles (for the wages $(w_E^1, w_E^2) = (0.25, 0.50)$) to be equally well off as in the household allocation $(l_E^1, l_E^2, Q_E) = (9.750, 7.315, 13.406)$. For our specification of the utility functions U^1 and U^2, we get

$$MMWI_E^1 = 11.433 \text{ and } MMWI_E^2 = 17.000,$$

We observe that the sum of $MMWI_E^1$ and $MMWI_E^2$ clearly exceeds the household income 19.5. Following Chiappori and Meghir (2014), this indicates gains from publicness of Q (and, thus, scale economies following from living together). In addition, we find that $MMWI_E^2 > MMWI_E^1$, suggesting a higher welfare of the more powerful individual 2. At this point, however, we must also emphasize that this kind of conclusions should be taken with sufficient caution. In particular, for different reference prices, one may well obtain a reverse ordering of the individual money metric welfare indices (see Chiappori and Meghir (2014) for more discussion and a graphical example). Finally, we note that, for the given reference prices, the difference between the individuals' $MMWI$s is less pronounced than between the income shares η_E^1 and η_E^2. This reflects the fact that household member 1 "benefits" from member 2's strong willingness to pay for the public consumption Q_E in the situation where the two individuals form a household.

By using the information contained by the Lindahl price sets Θ_n^1 and Θ_n^2 for the bundles $n \in N$, we can use the procedures presented in Section 4 to identify the upper bound $m_E^{i,ub}$ and lower bound $m_E^{i,lb}$. For our current application, this yields

$$m_E^{1,lb} = 8.050 \text{ and } m_E^{1,ub} = 13.681,$$
$$m_E^{2,lb} = 16.750 \text{ and } m_E^{2,ub} = 17.036.$$

Like before, we observe that the bounds $[8.050, 13.681]$ and $[16.750, 17.036]$ contain the true index values 11.433 and 17.000. Again, this confirms our theoretical results in Propositions 2 and 3. Next, our bounds are tight, in particular for individual 2. Intuitively, a higher bargaining weight combined with stronger preferences for public consumption, implies that the observed household behavior reveals more information on individual 2's preferences for the public good. In turn, this leads to tighter money metric bounds for this individual.

Importantly, our nonparametric bounds are also informatively tight. For example, they accurately reveal the household's gains from public consumption. We can identify these scale economies by comparing the sum $MMWI_E^1 + MMWI_E^2$ to the household expenditures y. In our case, even when we use the "conservative" lower bound estimates for the money welfare indices, we find a fairly large difference between $m_E^{1,lb} + m_E^{2,lb}$ $(= 24.8)$ and the household income y $(= 19.5)$, thus revealing substantial economies of scale associated with living together. Next, our bounds also correctly recover that member 2 achieves a higher money metric welfare than member 1 for the chosen reference prices: the sets $[8.050, 13.681]$ and $[16.745, 17.036]$ do not overlap, which means that the difference between $MMWI_E^1$ and $MMWI_E^2$ is clearly identified.

6 Empirical application

We show the practical usefulness of our method through an empirical application to data drawn from the 1999-2009 Panel Study of Income Dynamics (PSID). In particular, we consider a sample of 865 two-person households without children and for which both adult household members are participating in the labor market.[10] This data set was also studied by Cherchye, De Rock, Lewbel, and Vermeulen (2015) and we refer to this paper for additional details on the data construction method and sample selection procedure. In their original set-up, Cherchye, De Rock, Lewbel and Vermeulen distinguished between food, housing and other non-leisure expenditures. In line with our exposition in the previous sections, we treat all non-leisure consumption as a Hicksian (public) good.

Table 1 provides summary statistics on the relevant data for the sample at hand. Wages are net hourly wages. Leisure and annual hours worked are measured in hours per year. Full income and consumption expenditures are measured in nominal dollars per year.

	Mean	Std. Dev.	Min	Max
Male wage	28.43	18.82	3.43	140.77
Female wage	22.61	14.26	3.13	113.90
Male leisure	3,611.48	503.11	327.00	5,537.00
Female leisure	4,109.21	502.83	2,077.60	5,771.20
Male annual hours worked	2,212.52	503.11	287.00	5,497.00
Female annual hours worked	1,714.79	502.83	52.80	3,746.40
Expenditure on male leisure	103,502.04	72,549.92	3,569.24	567,204.85
Expenditure on female leisure	92,913.55	62,828.74	15,200.00	612,561.46
Expenditure on Hicksian good	39,463.99	24,404.09	8,200.00	183,716.00
Full income	235,879.59	117,467.87	75,620.37	716,813.60

Table 1: Summary statistics

To apply our method, we first estimate the household demand function g, which maps combinations of w^1, w^2 and y on $(l^1, l^2, Q) = g(w^1, w^2, y)$. Following Cherchye, De Rock,

[10] Labor supply decisions at the extensive margin imply complicated identification issues. See, for example, Donni (2003) and Blundell, Chiappori, Magnac, and Meghir (2007) for detailed discussions.

Lewbel, and Vermeulen (2015), this first stage reduces the level of 'noise' in the data. Second, it significantly increases the number of consumption bundles that the revealed preference approach can use to (set) identify individual income shares and money metric welfare indices. To operationalize the methods that we outlined in Sections 3 and 4, we draw a set N of demand bundles from the estimated demand. In the following application, we use $|N| = 400,000$.[11] We will show that this very rich set of consumption bundles obtains a significantly informative identification analysis. In our following exercises, we will consider a fully nonparametric demand system. The nonparametric regressions we use are Nadaraya-Watson kernel estimators with a (Gaussian) radial basis function kernel. This is a special case of the local polynomial estimator, for which it is known that the residuals are asymptotically normally distributed under certain regularity conditions (Fan and Gijbels, 1996).

To show the versatility of our method, in Appendix B we also discuss the results for a flexible parametric demand system. In particular, we consider Banks, Blundell, and Lewbel (1997)'s Quadratic Almost Ideal Demand System (QUAIDS). We show that, for our application, the nonparametric kernel-based results are close to the QUAIDS-based results. Given this close similarity between the nonparametric and parametric bounds, our following exposition will solely consider results that are based on the kernel estimation. An attractive feature of such a fully nonparametric analysis is that the empirical conclusions are very robust to functional specification error.

Two further remarks are in order. First, we do not impose Slutsky symmetry in our household demand estimations. Slutsky symmetry is needed for consistency with the unitary model but not for consistency with the collective model. Browning and Chiappori (1998) have shown that the collective model requires the existence of a household pseudo-Slutsky matrix that can be decomposed as the sum of a symmetric negative semi-definite matrix and a matrix of rank 1 (in the case of two household members, i.e. the so-called SR1 condition).[12] But this requirement needs at least five commodities to have empirical bite; it is trivially satisfied in our set-up with three commodities.[13] Second, our following analysis will not explicitly take into account estimation errors. Recent research has focused on inference for set identified objects. In particular, the studies of Kitamura and Stoye (2013), Henry and Mourifié (2013), Hoderlein and Stoye (2014) and Kaido, Molinari, and Stoye (2016) focus on set identification in a revealed preference context, and provide machinery that might be used for developing inference tools for our methodology. However, applying these techniques to our setting is quite a bit more com-

[11] More details on our drawing procedure are available upon request.

[12] See also Chiappori and Ekeland (2006) for related discussion.

[13] As indicated in Section 3 (see our discussion preceding Proposition 1), in our set-up a consumption observation E is not compatible with the model of collective rationality if $\theta_E^{i,lb} > \theta_E^{i,ub}$. Conveniently, for our data set we effectively obtain $\theta_E^{i,lb} \leq \theta_E^{i,ub}$ for every evaluated observation E, which means that our data satisfy this nonparametric consistency requirement for the collective rationality model.

plicated than existing applications to which such asymptotic theory has been successfully applied. Similar issues arise in other applications that combine demand estimation with revealed preference restrictions, such as Blundell, Browning, and Crawford (2008), Blundell, Kristensen, and Matzkin (2014) and Cherchye, De Rock, Lewbel, and Vermeulen (2015).

Sharing rule identification. As a first step, we compare our estimated upper and lower bounds for the individual income shares with so-called *naive* bounds. To construct these naive bounds, we exploit that leisure is private and assignable, so the value of a household member's leisure is a lower bound on that member's share of full income. This naive lower bound assigns all of the household's non-leisure consumption to the other household member. Similarly, a naive upper bound gives a household member his/her leisure and all of the household's non-leisure consumption. These naive bounds do not make use of any revealed preference restrictions associated with the collective household model. Comparing our revealed preference bounds with these naive bounds will provide insight into the identifying power of our identification method.

The results of this comparison are summarized in Table 2, which reports on the percentage point differences between upper and lower bounds on female expenditure shares for our sample of households. Since the individual shares sum up to one, the differences between the bounds are the same for men. Figure 4 in Appendix B gives an overview of the sharing rule results for our full sample of households. Comparing the naive bounds with the kernel-based bounds shows that our revealed preference based method provides a substantial improvement over the naive bounds, even with fully nonparametric demand function estimates. The average difference between the upper and lower naive bounds is about 17.52 percentage points, which narrows to 12.37 percentage points using the nonparametric estimates.

Interestingly, the bounds that we obtain are also informatively tight, which we illustrate in Table 3. This table reports on the distribution of the bounds for males and females in our sample. We report on the distribution of upper (resp. lower) bounds by showing deciles of the upper (resp. lower) bounds on individuals' expenditure shares. Generally, we find that male and female resource shares are increasing with income, which is of course fairly intuitive. Next, we also find that male shares are generally somewhat above the female shares. This reveals that households are frequently characterized by unequal resource sharing, which turns out to be mainly disadvantageous for females.

Recovery of money metric welfare indices. As argued in the Introduction, a specific advantage of our method over the method of Cherchye, De Rock, Lewbel, and Vermeulen (2015) is that we can identify individuals' money metric welfare indices. These indices are particularly well-suited for individual welfare analysis in the context of the

	naive bounds	kernel-based bounds
mean	17.52	12.37
minimum	3.28	2.53
1st quartile	12.12	8.72
median	15.78	11.20
3rd quartile	21.68	15.23
maximum	64.03	39.18
nr. obs.	865	865

Table 2: Percentage point differences between upper and lower bounds on individual female expenditure shares

	men		women	
percentile	lower bound	upper bound	lower bound	upper bound
10	44,164.1	62,357.1	37,762.5	56,460
20	57,650.8	77,453.9	51,661.7	69,787.1
30	69,100.9	90,366.2	60,266.8	80,947.2
40	78,354.6	99,948	69,939.2	92,845.2
50	88,799.1	114,435	82,624.6	109,464
60	103,382	130,149	94,371.4	121,275
70	120,469	152,908	107,762	139,339
80	148,955	183,500	134,420	172,288
90	195,878	244,181	172,784	221,761

Table 3: Sharing rule bounds

collective consumption model. We refer to Chiappori and Meghir (2014) for an in-depth discussion.

Table 4 gives a summary of the bounds on the money metric welfare indices that we obtain for our sample of households. Some interesting observations emerge from comparing the results in this table with the sharing rule results in Table 3. First, we find that the difference between the upper and lower bounds is generally close (and often tighter) in magnitude for the individual money metric indices than for the individual income shares. This shows that our method yields equally informative bounds for these two types of measures, which capture alternative dimensions of within-household inequality in consumption and welfare.

Next, although there is quite some overlap between the intervals, we observe that the bounds on the money metric indices are generally higher than for the individual resource shares (for both males and females). This indicates the cost of becoming single associated with the loss of public consumption (i.e. in a couple, public consumption is associated with individual Lindahl prices, whereas singles have to pay the (higher) market price for public consumption). Like for the expenditure shares in Table 3, the compensations required for males to achieve their within-household utility levels when they come to live alone are generally higher than the compensations required for females. Similarly to before, this reflects the unequal sharing of consumption within households (which was also captured by the sharing rule). However, the differences between the male and female money metric indices in Table 4 are not exactly the same as the differences between the male and female expenditure shares in Table 3. Intuitively, in terms of our structural model of collective household consumption, these discrepancies follow from diverging individual preferences for publicly consumed quantities.

	men		women	
percentile	$\mathrm{m}_E^{1,lb}$	$\mathrm{m}_E^{1,ub}$	$\mathrm{m}_E^{2,lb}$	$\mathrm{m}_E^{2,ub}$
10	52,895.3	69,890.8	43,251.8	63,479.4
20	69,541.5	82,515.9	56,974	77,481.9
30	83,638.5	95,415.5	71,217.5	89,660.5
40	96,506	105,755	81,567.8	100,327
50	108,569	118,185	97,669.7	112,620
60	122,383	132,816	111,427	126,840
70	138,957	151,361	128,192	146,763
80	173,670	190,340	154,572	174,757
90	220,701	243,533	207,386	229,784

Table 4: Bounds on money metric welfare indices

By using the results that are summarized in Table 4, we can analyze households' scale economies that follow from public consumption. Following again our conservative procedure, we obtain a lower bound estimate of households' gains by subtracting the current household income (y) from the sum of the nonparametrically estimated lower bounds on the individuals' money metric welfare indices ($m_E^{1,lb} + m_E^{2,lb}$). Figure 1 presents the distribution of these differences across our sample of households. We obtain non-positive differences for about 40 percent of the households. For these households, the current income does not exceed the sum of our estimated lower bounds on the individual $MMWI_E^i$ and, therefore, we cannot reject the hypothesis that there are no gains from public consumption. However, and more interestingly, for a large majority of our households, our conservative procedure does reveal strictly positive gains, again showing the informative value of our nonparametric identification method. As a matter of fact, for about one half of the households we learn that the gains from living together amounts to at least 10,000 dollars, which represents a significant fraction of the expenditures on the Hicksian public good for a modal household (see Table 1).

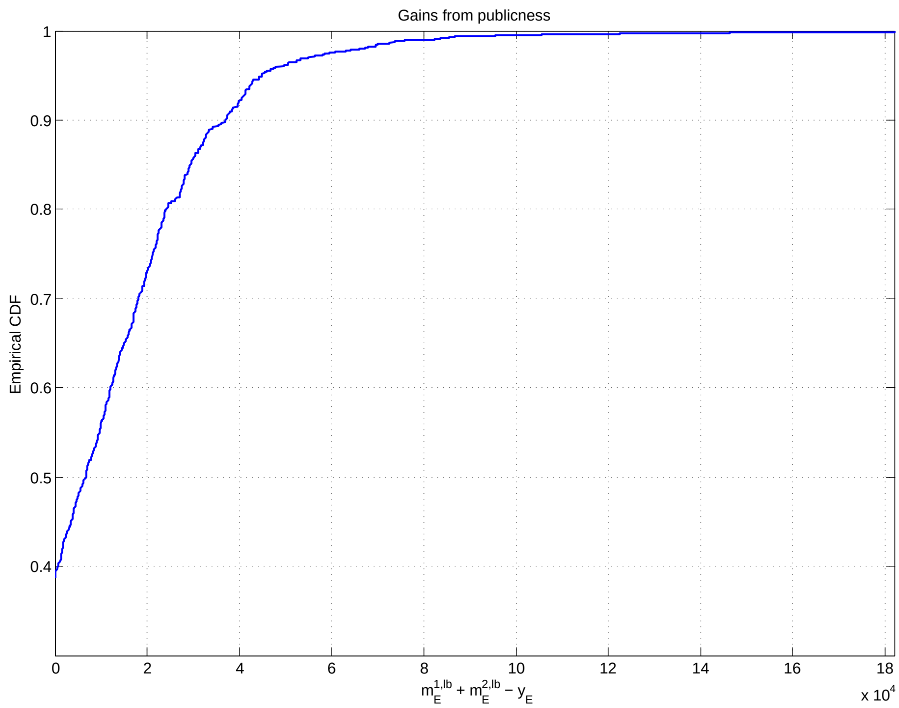


Figure 1: Gains from public consumption

Money metric welfare, total income and relative wages. The results of the above identification analyses can be used to address a variety of empirical questions that specifically relate to the intrahousehold distribution of consumption. For example, they allow one to analyze the effects of household characteristics like income and relative wages on individual consumption shares.[14] A specific feature of our method is that we can now also address these questions for money metric welfare indices.

[14]These relationships received considerable attention in the literature on collective consumption models. It is frequently assumed in the empirical literature that bargaining power is independent of total

Panel (a) of Figure 2 shows the relationship between the female money metric welfare indices, which we here express as proportions of the households' full incomes, and the logarithms of these households' full incomes. Each dot and plus sign on the figure represents the upper and lower bound for a given household in our sample. To help visualize the results, we include trendlines of the estimated upper and lower bounds.

The trendlines are slightly decreasing, but quite close to horizontal. This finding suggests that the female's money metric index (as a proportion of the household's full income) does not vary with total income. The trendlines in Figure 2 show that the average upper bounds are steadily around 55-65 percent and the average lower bounds around 45-50 percent. This implies that the female money metric welfare is between 45-50 percent (lower bound) and 55-65 percent (upper bound) of the total household income, on average. Note that the sum of male and female money metric welfare indices may well exceed the household income. Moreover, the figure also shows considerable heterogeneity across households. For example, some households have upper and lower bounds of the female money metric utility index around 90 percent, whereas other households have bounds around 10 percent.

Let us then compare these results to the ones of Cherchye, De Rock, Lewbel, and Vermeulen (2015), which are based on the sharing rule. These authors find that, on average, the female's (relative) income share is largely independent to the household's full income. Based on panel (a) of Figure 2, we can add that this independence conclusion also holds when using money metric welfare indices instead of the sharing rule. Next, a notable difference is that they find that the average female income share is situated between 40 percent (lower bound) and 50 percent (upper bound), whereas our average money metric indices are between 45-50 and 55-60 percent of the households' full incomes. Intuitively, this difference can be explained by the fact that the money metric indices take account of scale economies for public consumption, as we discussed above. Of course, given the different welfare-economic interpretation of the alternative concepts, these differences do not necessarily tell us much more.

Next, Panel (b) of Figure 2 shows the relationship between the female (relative) money metric indices and the relative wages. We clearly observe that a woman's money metric welfare index, as a proportion of the household's full income, generally increases when her relative wage goes up. Again, this conclusion concurs with that of the literature. It supports the argument that a household member's bargaining power generally increases with her/his wage, which results in higher individual welfare. In Appendix C, we show that this positive relationship is driven by a change in bargaining power, rather than

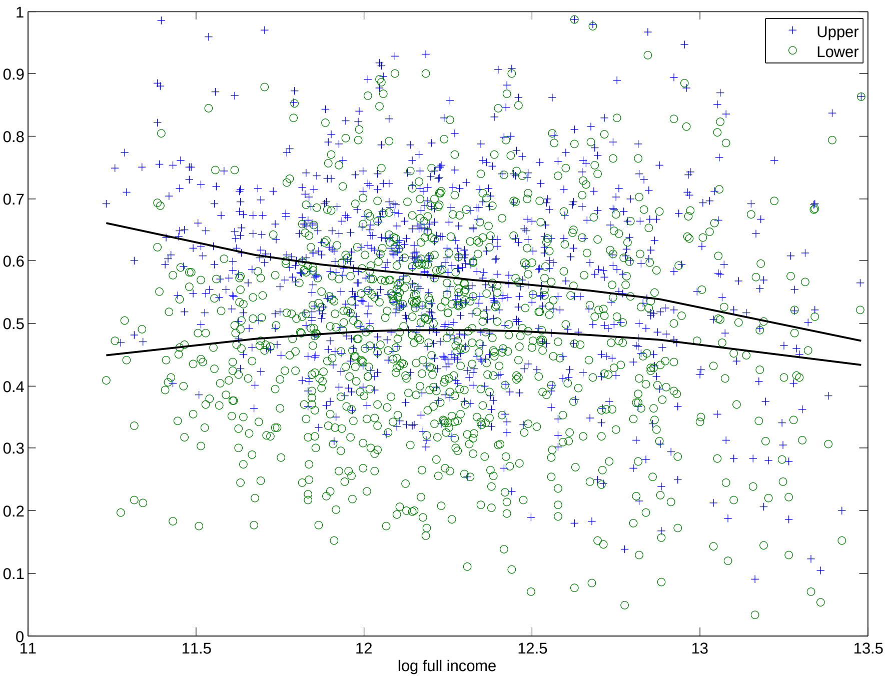
household income. See, for example, Lewbel and Pendakur (2008), Bargain and Donni (2012) and Dunbar, Lewbel, and Pendakur (2013), who use this assumption to obtain point identification for resource shares. Next, the literature also provided systematic evidence that a household member's bargaining power generally increases with her/his wage. See, for example, Chiappori, Fortin, and Lacroix (2002), Blundell, Chiappori, Magnac, and Meghir (2007) and Oreffice (2011).

a mechanical consequence of including leisure (evaluated at wages) in the individual resource shares.

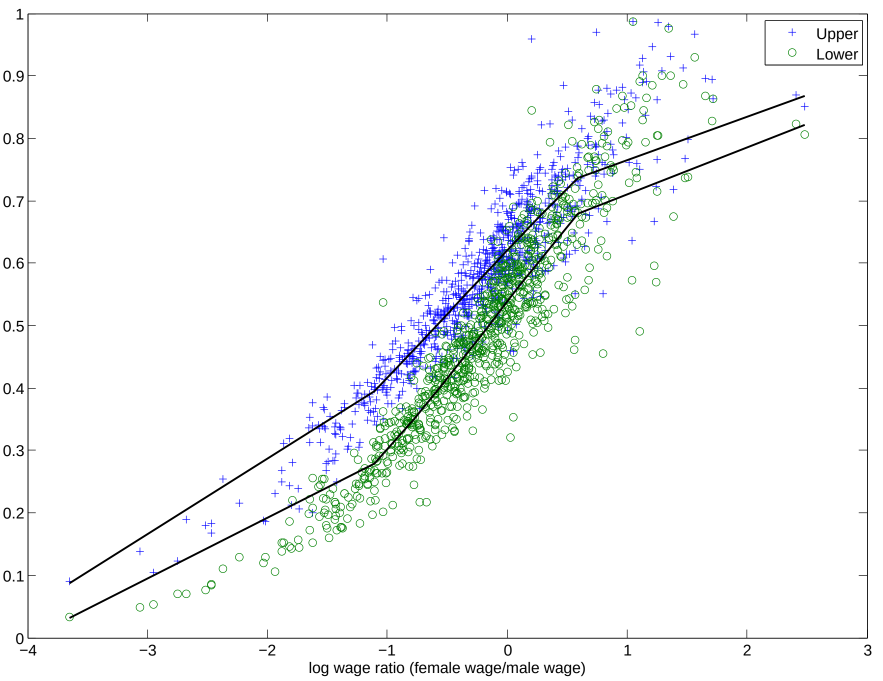
Individual poverty analysis. To conclude, our estimates allow us to conduct a poverty analysis directly at the level of individuals in households rather than at the level of aggregate households. By using the money metric indices, such a poverty analysis can simultaneously account for both economies of scale in consumption (through public goods) and within-household sharing patterns (reflecting individuals' bargaining positions). To clearly expose the impact of these two mechanisms, we perform three different exercises. In our first exercise, we compute the poverty rate defined in a more standard way, i.e. as the percentage of households having full income that falls below the poverty line, which we fix at 60 percent of the median full income in our sample of households. This also equals the individual poverty rates if there would be equal sharing and no economies of scale. The results of this exercise are given in Table 5 under the heading "Household poverty rate". We would label 11.33 percent of the individuals (and couples) as poor if we ignored scale economies and assumed that household resources are shared equally between males and females.

In a following exercise, we conduct an individual poverty analysis on the basis of the sharing rule. Here, we label an individual as poor if his/her income share estimate falls below the individual poverty line, which we define as half of the poverty line for couples that we used above. Based on our sharing rule bounds, we can compute upper and lower bound estimates for the individual poverty rates. The outcomes are summarized under the heading "Sharing rule" in Table 5. Our results indicate that, due to unequal sharing of resources within households, the fraction of individuals living below the poverty line may be considerably greater than the fraction obtained by standard measures that ignore intrahousehold allocations. In other words, the incidence of poverty at the individual level may be substantially higher than is indicated by standard measures based on household level income. In particular females in households appear to be at risk of poverty because of unequal resource sharing: even the lower bound estimate of female poverty (12.83 percent) is above the individual poverty rate that would occur in the case of equal sharing (11.33 percent, i.e. the household poverty rate).

Finally, we redid the individual poverty analysis (using the same poverty line) but now using the money metric welfare indices as the basis of our calculations. The results of this exercise are reported under the heading "MMWI" in Table 5. We find that both the lower and upper bound estimates of the individual poverty rates decrease when compared to the poverty results based on the sharing rule. Intuitively, the presence of public consumption (giving rise to scale economies) mitigates the risk of poverty. This clearly highlights the importance of households' scale economies in assessing individual poverty. For some households/individuals, publicness of consumption may partly offset



(a) female money metric welfare indices (lower bound $m_E^{2,lb}$ and upper bound $m_E^{2,ub}$) as proportions of the full household income (on vertical axis) versus log of full household income (horizontal axis)



(b) female money metric welfare indices (lower bound $m_E^{2,lb}$ and upper bound $m_E^{2,ub}$) as proportions of the household full household income (on vertical axis) versus log of wage ratio (on horizontal axis)

Figure 2: Female money metric welfare indices, full household income and wage ratio. The lines are trend lines obtained from a local linear fit.

the negative effect of unequal sharing and/or different individual Lindahl prices within the household. Our method effectively allows us to disentangle the impact of the two channels.

		Households	All individuals	Males	Females
Household poverty rate		11.33%	-	-	-
Sharing rule	Upper bound	-	26.71%	22.31%	31.10%
	Lower bound	-	10.69%	8.55%	12.83%
MMWI	Upper bound	-	18.44%	14.91%	21.97%
	Lower bound	-	6.24%	4.62%	7.86%

Table 5: Poverty rates

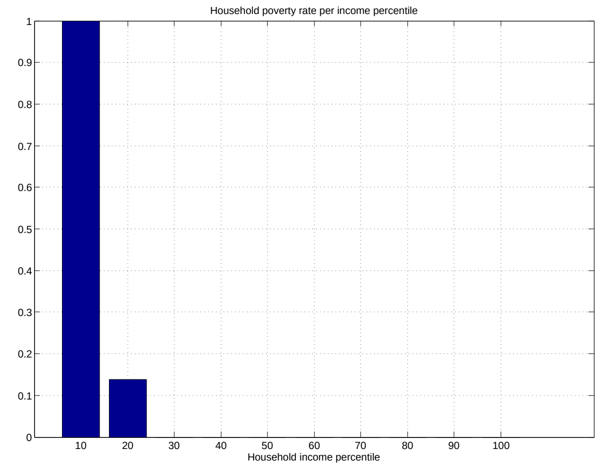
Figure 3 shows, for the different income deciles, the proportion of households and individual household members considered to be poor. Similar to before, the household poverty rate (panel (a)) is based on the household full income, while the bounds for the individual poverty rates are based on the bounds of the sharing rule (panel (b)) and the money metric welfare indices (panel (c)).

From panel (a) in Figure 3, we learn that the poor (aggregate) households are all situated in the bottom (20%) of the household income distribution. More revealingly, while households in lower income deciles are typically characterized by a higher degree of individual poverty, from panels (b) and (c) of the figure we also observe that poor individuals are actually situated along the entire income distribution (and even in the top deciles). Once more, this highlights the need to account for unequal resource sharing when evaluating individual poverty. Finally, panel (c) again shows that individual poverty rates (upper and lower bounds) are generally lower when based on money metric welfare indices, which means that we account for intrahousehold scale economies.

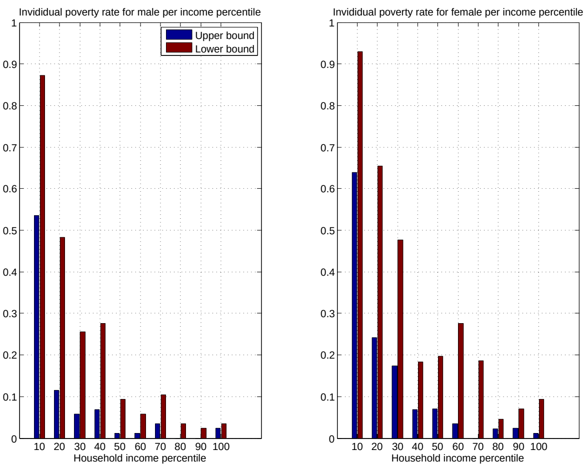
7 Conclusion

We have presented a novel empirical method to analyze individuals' welfare in a collective consumption setting. Our method allows us to set identify the intrahousehold sharing rule and individual money metric welfare indices from the observed household demand behavior. The method builds on a revealed preference characterization of the collective model that is intrinsically nonparametric. The method can be combined with nonparametric as well as parametric demand estimation. The possibility to conduct a fully nonparametric analysis is particularly attractive, as it yields empirical conclusions that are robust to functional specification error.

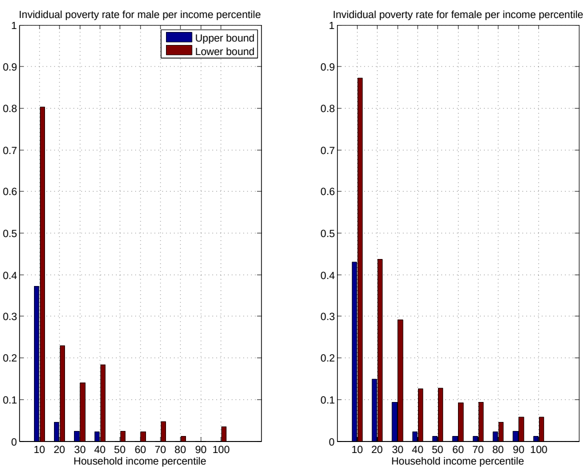
We have demonstrated the practical usefulness of our method through a simulation analysis and an empirical application to labor supply data drawn from the Panel Study of Income Dynamics (PSID). We showed that our nonparametric method obtains informa-



(a) household poverty



(b) individual poverty based on the sharing rule



(c) individual poverty based on the money metric welfare indices

Figure 3: Poverty rates at different income percentiles

tive bounds on the intrahousehold distribution of individual resource shares, willingness-to-pay for public consumption (i.e. Lindahl prices) and money metric welfare indices. In addition, our method clearly identified gains from public consumption (i.e. scale economies) associated with living together (versus living alone). Finally, we illustrated the potential of our method to investigate the effects of household characteristics like household income and relative wages on individual welfare, and to assess the incidence of individual (instead of household) poverty evaluated in terms of money metric welfare indices.

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A Proofs

Proof of Proposition 1. We will focus on $\theta_E^{i,ub}$, but a readily similar argument holds for $\theta_E^{i,lb}$. If $\min_{n\in\Omega_E^i, Q_n<Q_E} \frac{w_E^i(l_n^i - l_E^i)}{(Q_E - Q_n)} \geq 1$, then $\theta_E^{i,ub}$ is set equal to one and there is nothing to prove. So let us assume the reverse: there exists a bundle $n \in \Omega_E^i$ with $Q_n < Q_E$ such that $\theta_E^{i,ub} = \frac{w_E^i(l_n^i - l_E^i)}{(Q_E - Q_n)} < 1$. Given $n \in \Omega_E^i$, the following inequalities must hold:

$$\begin{aligned} w_n^i(l_E^i - l_n^i) &< 0, \qquad (8)\\ w_n^i(l_E^i - l_n^i) + (Q_E - Q_n) &< 0. \qquad (9) \end{aligned}$$

We are now ready to demonstrate that $\theta_E^{i,ub} + \delta \notin \Phi_E^i$, with δ infinitely small but positive:

$$\begin{aligned} & w_E^i(l_n^i - l_E^i) + \left(\frac{w_E^i(l_n^i - l_E^i)}{(Q_E - Q_n)} + \delta\right)(Q_n - Q_E) \\ & = \delta(Q_n - Q_E) \\ & < 0. \qquad (10) \end{aligned}$$

The last line follows from the requirement $Q_n < Q_E$. Conditions (8), (9) and (10) generate a violation of the WARP condition (1) in the main text. As an implication, the shadow price $(\theta_E^{i,ub}+\delta)$ does not sustain collective rationality and, therefore, $\theta_E^{i,ub}+\delta \notin \Phi_E^i$. Finally, $\delta(Q_n - Q_E) < 0$ shows that the above inequalities also hold for higher levels of

δ. Thus, $\theta_E^{i,ub}$ effectively constitutes an upper bound on the shadow prices sustaining collective rationality that are contained in Φ_E^i.

Proof of Proposition 2. To obtain the wanted conclusion (i.e. $IB^i(l_E^i, Q_E) \subseteq B^i(l_E^i, Q_E)$), we need to show $(l_n^i, Q_n) \in B^i(l_E^i, Q_E)$ for any $(l_n^i, Q_n) \in IB^i(l_E^i, Q_E)$.

As a first step, $(l_n^i, Q_n) \in IB^i(l_E^i, Q_E)$ implies

$$(Q_n - Q_E) \leq 0 \text{ and } w_n^i(l_n^i - l_E^i) + \theta_n^{i,ub}(Q_n - Q_E) \geq 0.$$

Because $\theta_n^{i,ub} \geq \theta_n^i$ for any $\theta_n^i \in \Theta_n^i$, these two inequalities imply $w_n^i(l_n^i - l_E^i) + \theta_n^i(Q_n - Q_E) \geq 0$ for any $\theta_n^i \in \Theta_n^i$.

Next, because $\Phi_E^i \subseteq \Theta_n^i$, we thus have $w_n^i(l_n^i - l_E^i) + \bar{\theta}_n^i(Q_n - Q_E) \geq 0$ for any shadow price $\bar{\theta}_n^i \in \Phi_E^i$ that sustains collective rationality (following Proposition 1). Then, using a basic (WARP-based) revealed preference argument (see, for example, Varian (1982)), this last inequality implies $U^i(l_n^i, Q_n) \geq U^i(l_E^i, Q_E)$ or, equivalently, $(l_n^i, Q_n) \in B^i(l_E^i, Q_E)$.

Proof of Proposition 3. Suppose that $(l^i, Q) \notin OB^i(l_E^i, Q_E)$. This implies that there exists $n \in N$ such that

$$\begin{aligned}
w_E^i(l_E^i - l_n^i) + \theta_E^{i,lb}(Q_E - Q_n) &\geq 0 \text{ if } Q_n < Q_E, \text{ or} \\
w_E^i(l_E^i - l_n^i) + \theta_E^{i,ub}(Q_E - Q_n) &\geq 0 \text{ if } Q_n \geq Q_E, \\
& \text{and} \\
w_n^i(l_n^i - l^i) + \theta_n^{i,lb}(Q_n - Q) &> 0.
\end{aligned}$$

From the first two inequalities, we can conclude that $U^i(l_E^i, Q_E) \geq U^i(l_n^i, Q_n)$ for the given n. Like in our proof of Proposition 2, this last result follows from a basic (WARP-based) revealed preference argument (see, for example, Varian (1982)) and using $\Phi_E^i \subseteq \Theta_E^i$ (following Proposition 1, for Φ_E^i containing the shadow prices that sustain collective rationality).

In what follows, we will consider two cases: $Q_n \geq Q$ and $Q_n < Q$. For each case, we will obtain that $(l^i, Q) \notin \{(l^{i\prime}, Q') \in B^i(l_E^i, Q_E) | Q' \leq Q_E\}$, which gives the wanted conclusion (i.e. $\{(l^i, Q) \in B^i(l_E^i, Q_E) | Q \leq Q_E\} \subseteq OB^i(l_E^i, Q_E)$).

We begin by considering $Q_n \geq Q$. Then, from the above constraints, the inequality $w_n^i(l_n^i - l^i) + \theta_n^i(Q_n - Q) > 0$ holds for any $\theta_n^i \in \Theta_n^i$. Given this, a similar revealed preference argument as above yields $U^i(l_n^i, Q_n) > U^i(l^i, Q)$. Thus, $U^i(l_E^i, Q_E^i) > U^i(l^i, Q)$, which gives $(l^i, Q) \notin B^i(l_E^i, Q_E)$.

Next, we turn to $Q_n < Q$. For $l_E^i > l^i$, when using that we need only consider $Q \leq Q_E$, a directly similar argument as before obtains $(l^i, Q) \notin B^i(l_E^i, Q_E)$.

Thus, the only situation left to consider is $Q_n < Q$ and $l_E^i \leq l^i$. If $w_E^i(l_E^i - l^i) +$

$\theta_E^{i,lb}(Q_E - Q) > 0$, the same argument as before implies $U^i(l_E^i, Q_E^i) > U^i(l^i, Q)$ and, thus, $(l^i, Q) \notin B^i(l_E^i, Q_E)$.

As a final step, we show that we can exclude $w_E^i(l_E^i - l^i) + \theta_E^{i,lb}(Q_E - Q) \leq 0$ for $Q_n < Q$ and $l_E^i \leq l^i$. To see this, we first note that, because $w_n^i(l_n^i - l^i) + \theta_n^{i,lb}(Q_n - Q) > 0$ and $Q_n < Q$, we must have $l_n^i > l^i$. Because $l^i \geq l_E^i$, this implies $(l_n^i - l_E^i) > 0$. Next, for $U^i(l_E^i, Q_E) \geq U^i(l_n^i, Q_n)$, collective rationality requires $\theta_n^{i,lb}(Q_E - Q_n) + w_n^i(l_E^i - l_n^i) \geq 0$. Using $(l_n^i - l_E^i) > 0$, we can rephrase this as

$$\frac{Q_E - Q_n}{l_n^i - l_E^i} \geq \frac{w_n^i}{\theta_n^{i,lb}}.$$

Recall that we do not consider the setting where $\theta_n^{i,lb} = 0$ or $\theta_E^{i,lb} = 0$, since this complies with an uninformative lower bound. Then, $w_n^i(l_n^i - l^i) + \theta_n^{i,lb}(Q_n - Q) > 0$ implies $Q < Q_n + \frac{w_n^i}{\theta_n^{i,lb}}(l_n^i - l^i)$. Using the above inequality, we can rewrite this as

$$Q < Q_n\left(\frac{l^i - l_E^i}{l_n^i - l_E^i}\right) + \frac{Q_E}{l_n^i - l_E^i}(l_n^i - l^i).$$

In turn, because $w_E^i(l_E^i - l_n^i) + \theta_E^{i,lb}(Q_E - Q_n) \geq 0$ implies $Q_n \leq \left(\frac{w_E^i(l_E^i - l_n^i)}{\theta_E^{i,lb}} + Q_E\right)$, we must have

$$Q < \left(\frac{w_E^i(l_E^i - l_n^i)}{\theta_E^{i,lb}} + Q_E\right)\left(\frac{l^i - l_E^i}{l_n^i - l_E^i}\right) + \frac{Q_E}{l_n^i - l_E^i}(l_n^i - l^i).$$

Rearranging obtains

$$0 < w_E^i(l_E^i - l^i) + \theta_E^{i,lb}(Q_E - Q),$$

which effectively excludes $w_E^i(l_E^i - l^i) + \theta_E^{i,lb}(Q_E - Q) \leq 0$.

B Additional empirical results

To show the versatility of our method, and to assess robustness of our main empirical results, we also combine our identification method with Banks, Blundell, and Lewbel (1997)'s Quadratic Almost Ideal Demand System (QUAIDS). This demand system was also used by Cherchye, De Rock, Lewbel, and Vermeulen (2015), and we refer to that paper for a detailed explanation of the QUAIDS system that we use here. We consider QUAIDS estimates with and without accounting for taste shifters (in casu, age of the husband and a dummy for home ownership). As in the main text, we consider a setting with three goods (i.e. leisure of the two spouses and Hicksian (public) consumption).

The QUAIDS-based sharing rule bounds are summarized in Table 6. Like Table 2, it reports on the percentage point differences between upper and lower bounds on individual expenditure shares for our sample of households. When comparing Tables 2 and 6, we find that the nonparametric kernel-based bounds are close to the QUAIDS-based bounds, which indicates that our parametric QUAIDS model yields results similar to the nonparametric demand system.

As a further comparison, Figures 4 and 5 present the width of the kernel-based and QUAIDS-based sharing rule bounds (in absolute terms). Each dot represents a household. The *improved* width is on the vertical axis and the *naive* width on the horizontal axis. Interestingly, the width of the sharing rule bounds is reduced by (more than) half for a significant number of households. In line with our above conclusion, the kernel-based and QUAIDS-based results are very similar.

	QUAIDS	QUAIDS with taste shifters
mean	11.35	11.03
minimum	2.55	2.54
1st quartile	7.82	7.47
median	10.27	10.02
3rd quartile	14.27	13.59
maximum	36.69	36.61
nr. obs.	865	865

Table 6: Percentage point differences between upper and lower bounds on individual female expenditure shares

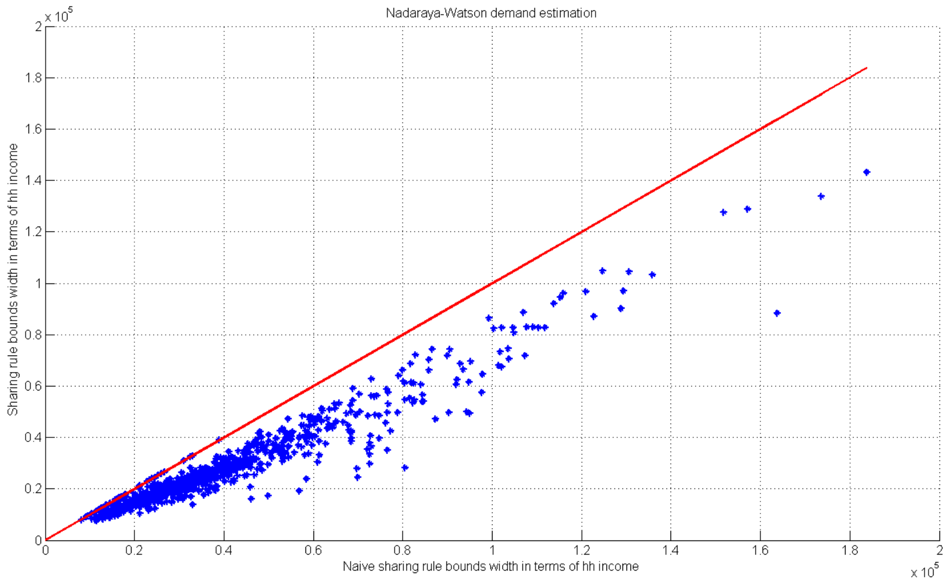

Figure 4: Width of sharing rule bounds in absolute terms (red line = naive bounds) (kernel-based)

Finally, Tables 7 and 8 have a directly similar interpretation as the kernel-based Tables 3 and 4 in the main text, but are QUAIDS-based (with and without taste shifters). Once

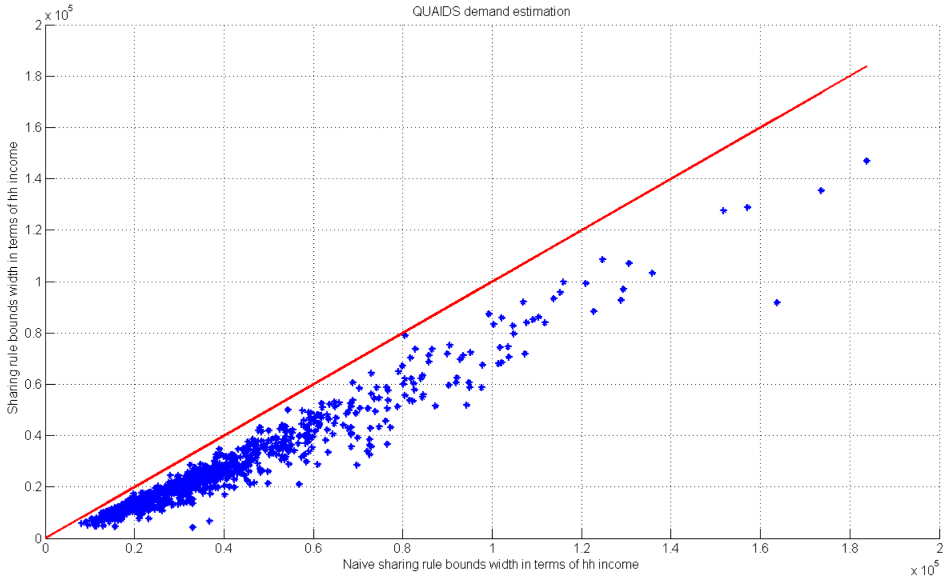

Figure 5: Width of sharing rule bounds in absolute terms (red line = naive bounds) (QUAIDS-based)

more, the QUAIDS-based analyses are very similar to the kernel-based results in the main text.

QUAIDS				
	men		women	
percentile	lower bound	upper bound	lower bound	upper bound
10	48,127.2	63,699.1	39,258.9	53,740.7
20	59,933.3	78,409.1	52,245.6	67,917.3
30	71,456.7	90,138.4	60,967.2	79,410.5
40	79,292.6	99,530.9	69,947.8	91,104.5
50	89,734.6	113,807	82,192.7	107,976
60	103,793	129,365	93,999.6	120,538
70	119,860	151,887	107,820	138,630
80	148,920	183,349	134,324	172,328
90	195,916	244,404	172,092	221,465
QUAIDS with taste shifters				
	men		women	
percentile	lower bound	upper bound	lower bound	upper bound
10	48,065.9	62,590.8	39,415.9	53,923.2
20	59,850.1	76,985.6	53,013.4	67,913.6
30	71,555.3	88,620.5	61,436.7	79,558.5
40	79,304	99,369.7	70,085.3	91,254.5
50	90,223.5	113,528	82,793.9	107,611
60	103,539	130,136	94,739.6	120,127
70	120,159	151,442	107,582	138,357
80	149,019	183,164	134,659	172,221
90	195,891	244,331	171,372	221,493

Table 7: Sharing rule bounds

QUAIDS				
	men		women	
percentile	$m_E^{1,lb}$	$m_E^{1,ub}$	$m_E^{2,lb}$	$m_E^{2,ub}$
10	57,866.7	67,204.5	41,192.2	60,832.9
20	74,983.4	82,650.8	55,874.9	75,505.5
30	86,557.7	94,561.7	67,554.7	88,240.5
40	97,474.5	104,772	77,208.6	98,075.9
50	108,503	117,949	91,258.5	113,365
60	122,881	135,949	104,105	127,954
70	140,977	154,790	121,206	143,848
80	173,473	188,494	152,268	177,347
90	224,826	246,064	199,745	223,227
QUAIDS with taste shifters				
	men		women	
percentile	$m_E^{1,lb}$	$m_E^{1,ub}$	$m_E^{2,lb}$	$m_E^{2,ub}$
10	56,820.7	65,082.6	42,185.1	57,418.7
20	72,988.3	80,004	55,455.9	70,841.8
30	84,717.7	90,896	67,619.4	83,612.9
40	96,075.3	101,555	76,738.2	94,335.6
50	106,164	114,491	90,677	109,169
60	120,662	131,311	103,578	123,526
70	139,229	150,138	120,500	138,481
80	170,960	180,885	150,672	171,849
90	221,360	239,860	198,239	218,258

Table 8: Bounds on money metric welfare indices

C Shadow prices and relative wages

To show that our results are not a mechanical consequence of including leisure in the individual resource share, we present the relationship between $\theta/(1-\theta)$ (with θ the male's shadow price for public consumption) and the wage ratio (female wage/male wage). We observe that the male's (relative) shadow price is generally decreasing in the female's relative wage. This shadow price does not contain leisure but essentially depends on the individuals' bargaining weights (for given individual preferences). The result indicates that the male's willingness-to-pay for public consumption tends to decrease in relative terms when the female's relative wage increases. Intuitively, it suggests that the intrahousehold public consumption complies more with the female's preferences when her relative wage increases.

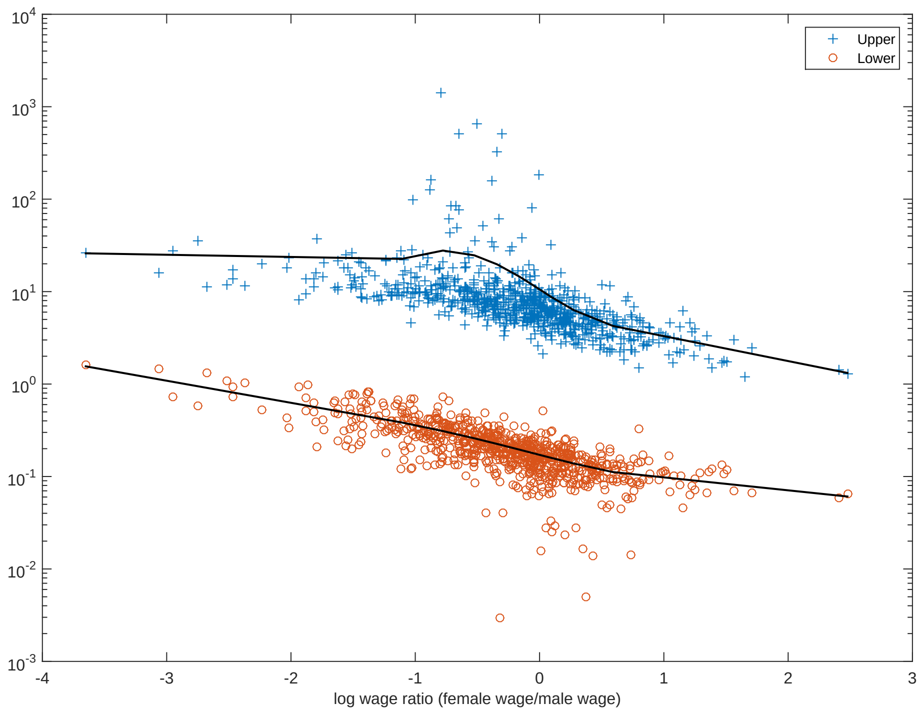

Figure 6: The ratio $\theta/(1-\theta)$ (on vertical axis; with θ the male's shadow price for public consumption) versus log of wage ratio (on horizontal axis)