

To mix or specialise? A coordination productivity indicator for English and Welsh farms*

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Abstract

This paper introduces a nonparametric measure of coordination productivity growth where the subprocesses are explicitly modelled in the production technology. The coordination productivity indicator is decomposed into a coordination technical inefficiency change component and a coordination technical change component. This decomposition allows assessment of re-allocation impacts on the different sources of productivity growth. The empirical application focusses on a large panel of English and Welsh farms over the period 2007 – 2013. The results show that coordination inefficiency significantly increases with the proportion of resources allocated to livestock production in economic and statistical terms. Coordination inefficient farms should generally allocate more land to crop production. Depending on the region, the average coordination productivity growth ranges from -9.7 percent to 15.9 percent per year. It is driven by coordination technical change rather than coordination inefficiency change.

Keywords: directional distance function, Luenberger productivity indicator, coordination inefficiency, mixed farms

JEL classification: D22, D24, Q12

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1 Introduction

The economic choice for a farmer about whether to engage in specialised or mixed agriculture is based on a comparison of the gains from economies of scale versus the gains from diversifying risk and economies of scope. Crop-specific capital (*e.g.* harvesters and ploughs) and livestock-specific capital (*e.g.* milking robots) are expensive and benefit considerably from economies of scale. Furthermore, these fixed costs of capital can be spread more over higher production volumes. Thus, a return on these investments can only be achieved by increasing the scale of operation, which in turn leaves little room for other farming activities (Chavas and Aliber, 1993; Fernandez-Cornejo et al., 1992). European agriculture is nowadays increasingly characterised by specialised production. In the light of the liberalisation of the Common Agricultural Policy, input and output prices are becoming more volatile, increasing the volatility of the farmer's income. Economic intuition suggests that diversifying into more farming activities and mixed farming generates economies of scope resulting from complementarities between different farming activities, which allows for production at lower cost (Chavas, 2008). Moreover, researchers and policy makers are increasingly concerned about the negative environmental impact of nutrient surplus associated with specialisation (Ryschawy et al., 2012).

The efficiency and productivity of a farm play an essential role for its long-term viability, requiring coordination of crop- and livestock-specific inputs. The overwhelming majority of the studies in the efficiency and productivity literature treat agricultural production as a black-box where the subprocesses are overlooked.¹ This implicitly leaves the question unanswered whether more or less specialisation would be needed for efficiency gains. In addition, this hampers the comparability of farms with different subprocesses. The difficulty of modelling these subprocesses may explain why most empirical studies only focus on specialised farms. We introduce a coordination Luenberger productivity indicator that addresses these problems.

Färe and Whittaker (1995) introduce an efficiency framework that takes into account the production of intermediate inputs on the farm. In their model, crop output can also be used as a feed input in the livestock enterprise. Focussing on a sample of cereal farms, Färe et al. (1997) develop an efficiency framework where land use can be reallocated. Cherchye et al. (2013) develop a general framework that opens the black-box of production by explicitly modelling input allocation in a multi-output setting. They distinguish different subdivisions with their own output. Every output uses its own associated output-specific inputs and common

¹Färe and Whittaker (1995), Färe et al. (1997), Jaenicke (2000), Skevas et al. (2012) and Chen (2012) are exceptions.

joint inputs that are shared by all outputs. They develop a radial input-oriented framework. Using this framework, [Cherchye et al. \(2015\)](#) address the question of efficient allocation of common output-specific inputs over subdivisions. They develop a coordination efficiency measure that quantifies the possible efficiency gains from reallocating some inputs over the subdivisions.

Using a nonparametric framework, this paper extends the static, radial framework suggested by [Cherchye et al. \(2015\)](#) to a dynamic context of coordination Luenberger productivity growth. [Cherchye et al. \(2015\)](#)'s radial framework only identifies inefficiencies in the input direction. We generalise this approach by developing a directional distance function framework where inputs as well as outputs are choice variables, which is consistent with profit-maximising behaviour. The Luenberger productivity indicator builds on contemporaneous and intertemporal directional distance functions. It measures productivity growth by simultaneously assessing changes in the input and output level over time and can be decomposed into components of technical inefficiency change and technical change. Explicitly taking into account the subprocesses of crop production and livestock production, our framework is able to adequately compare the efficiency and productivity of crop farms, mixed farms and livestock farms. It also indicates whether coordination efficiency gains would be associated with specialisation or diversification towards mixed farming. Moreover, we define a coordination productivity indicator that measures productivity growth due to optimal reallocation of process-specific inputs over time which is decomposable in a coordination technical inefficiency change component and a coordination technical change component. This decomposition allows us to assess how reallocation affects the different sources of productivity growth. The empirical application focusses on panel data from mixed and specialised farms in England and Wales over the period 2007 – 2013.

The remainder of the paper is structured as follows. The next section describes the theoretical framework for measuring the coordination Luenberger productivity indicator and its components. This is followed by the practical implementation and empirical application. The final section concludes.

2 Mixed farm model

In this section we describe our mixed farm model. We distinguish two interdependent subprocesses with their own technology. We then propose a coordination Luenberger productivity indicator and its decomposition that identifies how coordination inefficiency affects the different sources of productivity growth.

2.1 Model and technology description

We identify 2 processes: the crop subprocess (C) and the livestock subprocess (L). The network structure is shown in Figure 1. Following Färe and Whittaker (1995), both processes are interdependent because the livestock process uses unmarketed residue of crops as feed for its livestock in addition to feed bought on the market. Note that manure could be modelled as a livestock output, which can serve as an input for future crop production. However, we do not include manure in our model due to a lack of availability of manure data.²

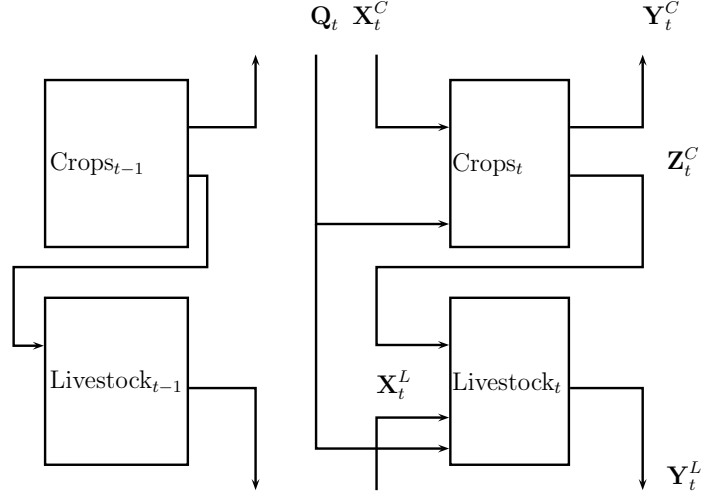


Figure 1: Network structure of the model

The crop subprocess has the following inputs and outputs:

- $\mathbf{X}_t^C \in \mathbb{R}_+^{N_C}$: inputs such as labour, seeds, etc;
- $\mathbf{Y}_t^C \in \mathbb{R}_+^{O_C}$: outputs such as wheat, barley, etc;
- $\mathbf{Z}_t^C \in \mathbb{R}_+^{O_C}$: outputs that are not sold, but used as feed in the same period for the livestock.

The livestock subprocess has the following inputs and outputs:

- $\mathbf{X}_t^L \in \mathbb{R}_+^{N_L}$: inputs such as labour, feed, etc;
- $\mathbf{Z}_t^C \in \mathbb{R}_+^{O_C}$: outputs from the crops used as feed in the same period;

²At the time of writing, manure data were only available for the years 2012 and 2013 for a subsample of farms.

- $\mathbf{Y}_t^L \in \mathbb{R}_+^{O_L}$: outputs such as milk, meat, etc;

Note that \mathbf{X}_t^L and \mathbf{Z}_t^C can have common inputs: the farmer buys additional feed for his livestock on top of the feed he already collected from his crops. Define the index set $H = \{1, \dots, N_L\} \cap \{1, \dots, O_C\}$ for these common inputs.

There can be some inputs that are shared by both processes but which are not joint. Land use is such an input: the farmer has to decide how much of his land area to use for crop production and livestock production. Thus, these subprocess-specific inputs have to be allocated among both processes. In line with Cherchye et al. (2015), this allocation might not be optimal and a better reallocation is possible. Let $\mathbf{X}_t \in \mathbb{R}_+^C$ with $C \subseteq \{1, \dots, N_C\} \cap \{1, \dots, N_L\}$ be the process-specific inputs that have to be allocated among both subprocesses such that

$$X_t^{C,m} + X_t^{L,m} = X_t^m \quad \forall m \in C. \quad (1a)$$

Thus, C is the subset of inputs, common to crop and livestock, that can be reallocated among both subprocesses. In line with Färe et al. (1997), this application regards crop land and livestock land as reallocatable, fixed inputs.

Furthermore, the levels of the other process-specific inputs must also be adjusted for the new reallocation:

$$\sum_{\forall i \in \{1, \dots, N_C\} \setminus C} p_t^{C,i} X_t^{C,i} + \sum_{\forall j \in \{1, \dots, N_L\} \setminus C} p_t^{L,j} X_t^{L,j} = PEX P_t, \quad (1b)$$

where $\mathbf{p}_t^C \in \mathbb{R}_{++}^{N_C}$ and $\mathbf{p}_t^L \in \mathbb{R}_{++}^{N_L}$ are the prices of crop-specific and livestock-specific inputs and $PEX P_t$ is the total process-specific expenditures.³ This budget constraint allows the farmer to redistribute the process-specific budget over the crop and livestock activities while staying within his budget. In general, we do not know the individual farmer's credit or budget constraints, but we do observe his total process-specific expenditures. Therefore, process-specific expenditures can be reallocated within the farmer's observed budget.

Finally, the crop and livestock process share a joint input $\mathbf{Q}_t \in \mathbb{R}_+^M$ (*e.g.* buildings and machinery). Joint inputs are inputs which are shared by the different subprocesses (see Cherchye et al. (2013)). Some of these joint inputs are fixed: let $F \subseteq \{1, \dots, M\}$ denote the set of fixed joint inputs.

We now define the technology of each subprocess by their production set. The crop subprocess production set is:

$$\mathcal{Y}_t^C = \{(\mathbf{X}_t^C, \mathbf{Q}_t) \text{ produces } (\mathbf{Y}_t^C, \mathbf{Z}_t^C)\}. \quad (2)$$

³In the absence of price data, one could equivalently work with expenditures. The quantities are then expenditures and the modified budget constraint would be (1b) without prices.

Similarly, the livestock subprocess production set is:

$$\mathcal{Y}_t^L = \{(\mathbf{X}_t^L, \mathbf{Z}_t^C, \mathbf{Q}_t) \text{ produces } \mathbf{Y}_t^L\}. \quad (3)$$

In the remainder of this paper, we assume the following basic axioms for both subprocesses:

Axiom 1 (strong disposability of inputs). $(\mathbf{x}, \mathbf{y}) \in \mathcal{Y}$ and $\mathbf{x}' \geq \mathbf{x} \implies (\mathbf{x}', \mathbf{y}) \in \mathcal{Y}$

Axiom 2 (strong disposability of outputs). $(\mathbf{x}, \mathbf{y}) \in \mathcal{Y}$ and $\mathbf{y}' \leq \mathbf{y} \implies (\mathbf{x}, \mathbf{y}') \in \mathcal{Y}$

Axiom 3 (convexity). *Technology set \mathcal{Y} is convex.*

The overall network production set is:

$$\mathcal{Y}_t = \{(\mathbf{X}_t^C, \mathbf{Q}_t, \mathbf{Y}_t^C, \mathbf{Z}_t^C) \in \mathcal{Y}_t^C \text{ and } (\mathbf{X}_t^L, \mathbf{Z}_t^C, \mathbf{Q}_t, \mathbf{Y}_t^L) \in \mathcal{Y}_t^L\}, \quad (4)$$

and satisfies the above axioms by construction.

2.2 The Luenberger productivity indicator and its decomposition

We use Luenberger's directional distance function to measure technical inefficiency by simultaneously contracting inputs and expanding outputs. This is consistent with profit maximisation. Shephard (1970)'s input and output distance functions are special cases of the directional distance function (Chambers et al., 1996a) and are consistent with cost minimisation and revenue maximisation, respectively. Define, for notational convenience, $\mathbf{X}_t = (\mathbf{X}_t^C, \mathbf{X}_t^L, \mathbf{Z}_t^C, \mathbf{Q}_t)$ as the input vector and $\mathbf{Y}_t = (\mathbf{Y}_t^C, \mathbf{Y}_t^L, \mathbf{Z}_t^C)$ as the output vector. The directional distance function proposed by Chambers et al. (1996b) is:

$$D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t) = \sup \{\beta \in \mathbb{R} : (\mathbf{X}_t - \beta \mathbf{g}_{x,t}, \mathbf{Y}_t + \beta \mathbf{g}_{y,t}) \in \mathcal{Y}_t\}, \quad (5)$$

if $(\mathbf{X}_t - \beta \mathbf{g}_{x,t}, \mathbf{Y}_t + \beta \mathbf{g}_{y,t}) \in \mathcal{Y}_t$ for some β and $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t) = -\infty$ otherwise. Here, $\mathbf{g}_t = (\mathbf{g}_{x,t}, \mathbf{g}_{y,t})$ represents the direction vector. The directional distance function is a special case of Luenberger (1992)'s shortage function.

We denote the time-related directional distance function for $(a, b) \in \{t, t+1\} \times \{t, t+1\}$:

$$D_b(\mathbf{X}_a, \mathbf{Y}_a; \mathbf{g}_a) = \sup \{\beta \in \mathbb{R} : (\mathbf{X}_a - \beta \mathbf{g}_{x,a}, \mathbf{Y}_a + \beta \mathbf{g}_{y,a}) \in \mathcal{Y}_b\}.$$

Furthermore, we distinguish between $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t | R)$ and $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t | NR)$: the former allows for reallocation of the process-specific inputs over the subprocesses (i.e. (1)), while the latter keeps these fixed.

In an analogous way, we define “reallocative” and “non-reallocative” Luenberger productivity indicators $L_{t,t+1}(\cdot|R)$ and $L_{t,t+1}(\cdot|NR)$ respectively. The Luenberger productivity indicator proposed by [Chambers \(2002\)](#) is defined as:

$$\begin{aligned} L_{t,t+1}(\mathbf{X}_t, \mathbf{Y}_t, \mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_t, \mathbf{g}_{t+1}|c) \\ = \frac{1}{2} [(D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|c) - D_t(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1}|c)) \\ + (D_{t+1}(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|c) - D_{t+1}(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1}|c))] , \end{aligned} \quad (6)$$

for $c \in \{R, NR\}$. It can be additively decomposed into a technical inefficiency change component and a technical change component:

$$\begin{aligned} L_{t,t+1}(\cdot|c) &= (D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|c) - D_{t+1}(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1}|c)) \\ &+ \frac{1}{2} [(D_{t+1}(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1}|c) - D_t(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1}|c)) \\ &+ (D_{t+1}(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|c) - D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|c))] \\ &\equiv \Delta TEI(c) + \Delta T(c), \end{aligned} \quad (7)$$

where the first difference is the technical inefficiency change component $\Delta TEI(c)$ and the arithmetic average of the two last differences captures technical change $\Delta T(c)$ ([Chambers et al., 1996b](#)). The technical inefficiency change component quantifies the change in relative position of a given observation to the (shifted) production frontier. The technical change component measures the change in the production frontier itself and is therefore a measure of technical progress or regress. The relevant distance functions are depicted in Figure 2.

2.3 Coordination inefficiency

[Cherchye et al. \(2015\)](#) consider a model of a Decision Making Unit with several subdivisions (they give an example of a university with subdivisions in research and teaching). They are interested in measuring whether efficiency gains are possible from reallocating common inputs over the different subdivisions. To this end, they distinguish radial measures of decentralised and centralised efficiency. Decentralised efficiency is the radial measure of efficiency when the current allocation is preserved over the subdivisions. In contrast, centralised efficiency is the radial measure of efficiency when the allocation is free to change over the subdivisions. Then, they define coordination efficiency as the ratio of centralised over decentralised efficiency. We make use of a directional distance function framework, which is more flexible in that it allows for varying input and output levels.

An equivalent difference-based coordination inefficiency measure is:

$$CI = D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|R) - D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|NR). \quad (8)$$

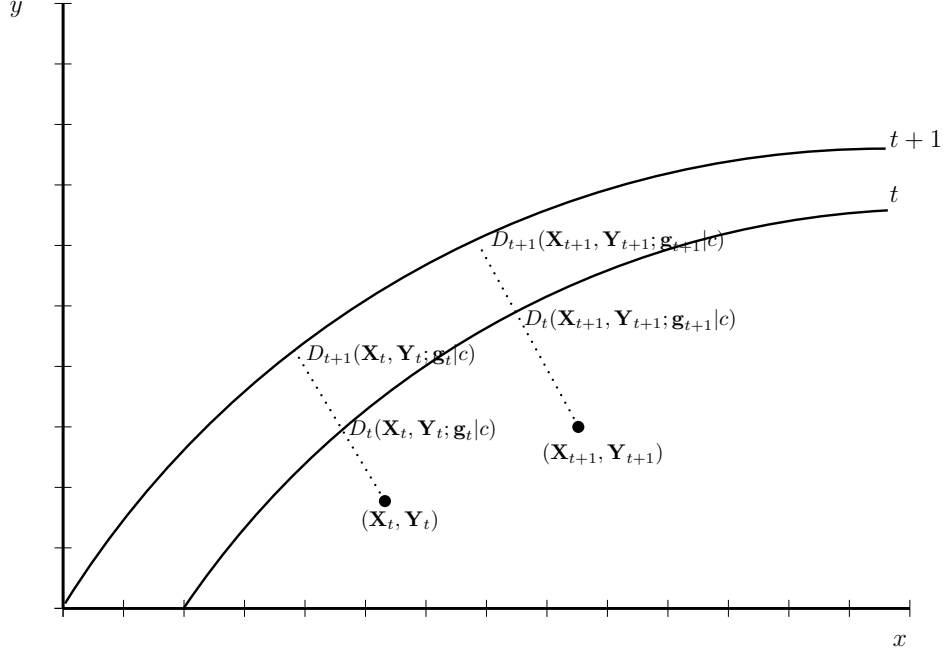


Figure 2: Distance functions of the Luenberger productivity indicator

where R, NR denote reallocation and no reallocation from present farm organisation, respectively. Here, one can see that $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|R) \geq D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|NR)$, because the *status quo* allocation represented by $D_t(\cdot|NR)$ is always attainable when reallocation is allowed. Positive values for CI indicate that inefficiencies may arise from suboptimal allocation of inputs.

2.4 Coordination productivity indicator

We measure how much productivity growth is affected by reallocation of inputs over time by comparing the reallocative Luenberger productivity indicator with the non-reallocative Luenberger productivity indicator. $L_{t,t+1}(R) > (<) L_{t,t+1}(NR)$ indicates that a farmer becomes better (worse) at reallocating over time which leads to improved (worsened) productivity growth. We define a “coordination Luenberger productivity indicator” $CL_{t,t+1}$ as the difference between $L_{t,t+1}(R)$ and $L_{t,t+1}(NR)$:

$$\begin{aligned} CL_{t,t+1} &\equiv L_{t,t+1}(R) - L_{t,t+1}(NR) \\ &= [\Delta TEI(R) - \Delta TEI(NR)] + [\Delta T(R) - \Delta T(NR)] \\ &\equiv \Delta CI + \Delta CT, \end{aligned} \tag{9}$$

where ΔCI is coordination inefficiency change and ΔCT is coordination technical change. ΔCI measures the change in coordination inefficiency that can be ascribed

to reallocation of process-specific inputs over time. ΔCT measures changes in the production frontier due to reallocation of process-specific inputs over time.

3 Practical implementation

The empirical analyst can compute efficiency and productivity measures using either a parametric or nonparametric approach. The parametric approach takes into account stochastic factors and does not treat all deviations from the frontier as inefficiency. However, it requires a specification of a functional form and technical changes cannot be determined at the firm level. We opt for the nonparametric approach which does not require such a specification and allows for determination of firm-specific technical changes ([Oude Lansink et al., 2015](#)).

We assume that we have data

$$S = \{\mathbf{p}_{k,t}^C, \mathbf{X}_{k,t}^C, \mathbf{p}_{k,t}^L, \mathbf{X}_{k,t}^L, \mathbf{Z}_{k,t}^C, \mathbf{Q}_{k,t}, \mathbf{Y}_{k,t}^C, \mathbf{Y}_{k,t}^L\}_{t=1}^T$$

for Decision-Making Unit (DMU) $k = 1, \dots, K$. The DMU under evaluation is $k = 0$.

3.1 Technology

The crop production set for a variable-returns-to-scale technology can be empirically approximated as:

$$\hat{\mathbf{y}}_t^C = \{(\mathbf{X}_{0,t}^C, \mathbf{Q}_{0,t}, \mathbf{Y}_{0,t}^C, \mathbf{Z}_{0,t}^C) : \sum_{k=1}^K \lambda_{k,t} \mathbf{X}_{k,t}^C \leq \mathbf{X}_{0,t}^C, \quad (10a)$$

$$\sum_{k=1}^K \lambda_{k,t} \mathbf{Q}_{k,t} \leq \mathbf{Q}_{0,t}, \quad (10b)$$

$$\sum_{k=1}^K \lambda_{k,t} (\mathbf{Y}_{k,t}^C + \mathbf{Z}_{k,t}^C) \geq (\mathbf{Y}_{0,t}^C + \mathbf{Z}_{0,t}^C), \quad (10c)$$

$$\sum_{k=1}^K \lambda_{k,t} = 1, \quad (10d)$$

$$\lambda_{k,t} \geq 0 \}. \quad (10e)$$

The livestock production set for a variable-returns-to-scale technology can be empirically approximated as:

$$\hat{\mathbf{y}}_t^L = \{(\mathbf{X}_{0,t}^L, \mathbf{Z}_{0,t}^C, \mathbf{Q}_{0,t}, \mathbf{Y}_{0,t}^L) : \sum_{k=1}^K \gamma_{k,t} X_{k,t}^{L,h} \leq X_{0,t}^{L,h} \forall h \notin H, \quad (11a)$$

$$\sum_{k=1}^K \gamma_{k,t} (Z_{k,t}^{C,h} + X_{k,t}^{L,h}) \leq Z_{0,t}^{C,h} + X_{0,t}^{L,h} \forall h \in H, \quad (11b)$$

$$\sum_{k=1}^K \gamma_{k,t} \mathbf{Q}_{k,t} \leq \mathbf{Q}_{0,t}, \quad (11c)$$

$$\sum_{k=1}^K \gamma_{k,t} \mathbf{Y}_{k,t}^L \geq \mathbf{Y}_{0,t}^L, \quad (11d)$$

$$\sum_{k=1}^K \gamma_{k,t} = 1, \quad (11e)$$

$$\gamma_{k,t} \geq 0 \}. \quad (11f)$$

These approximations are the inner bound approximations of the technology (Varian, 1984). From these subprocesses we obtain the approximation $\hat{\mathbf{y}}_t$ of the overall technology by taking the intersection of $\hat{\mathbf{y}}_t^C$ and $\hat{\mathbf{y}}_t^L$.

3.2 Inefficiency measurement

The implementation of the directional distance function (5) is:

$$D_t(\mathbf{X}_{0,t}, \mathbf{Y}_{0,t}; \mathbf{g}_t) = \sup \left\{ \beta \in \mathbb{R} : (\mathbf{X}_{0,t} - \beta \mathbf{g}_{x,t}, \mathbf{Y}_{0,t} + \beta \mathbf{g}_{y,t}) \in \hat{\mathbf{y}}_t \right\}. \quad (12)$$

The combination of subprocesses is implemented by solving linear programme (B1) which essentially combines (10), (11) and (12). The linear programmes are relegated to the online Appendix to conserve space. In line with the literature (*e.g.* Chambers et al. (1996b)), we select $\mathbf{g}_{x,t} = \mathbf{X}_t$ and $\mathbf{g}_{y,t} = \mathbf{Y}_t$ as the directional vectors. This choice ensures that the contemporaneous directional distance function is feasible (Briec and Kerstens, 2009) and can be interpreted as the maximum proportional contraction of variable inputs and simultaneously as the maximum proportional expansion of outputs.

The directional distance function which allows for reallocation of land and the process-specific variable costs is computed by solving the linear programme (B2) in the online Appendix. Compared to (B1), the crop-specific ($\mathbf{X}_{0,t}^C$) and

livestock-specific ($\mathbf{X}_{0,t}^L$) variable inputs are additional choice variables in this linear programme. Furthermore, it has two additional constraints to ensure that (i) the sum of the optimal crop land and livestock land is equal to the total land area; (ii) the process-specific variable costs can be optimally redistributed over the crop and livestock activities. (ii) is in line with [Färe and Grosskopf \(2012\)](#)’s cost-constrained efficiency measure. Consider the following example to see why this redistribution of the process-specific variable costs is necessary: it would make little sense for a fully specialised livestock farm to diversify into crops without this reallocation of the budget, for he would not be able to buy the necessary seeds for his crop land (i.e. $X_{0,t}^{C,m} = 0$ in (B1a)). Therefore, he must be able to reallocate part of his budget to crop specific inputs (such that $X_{0,t}^{C,m} > 0$). An analogous reasoning holds for a fully specialised crop farm.⁴

Note that our model makes several implicit assumptions about land use. First, we assume that land is immediately reallocatable and costless.⁵ Second, we assume that all the farm’s utilised land is substitutable between crops and livestock and thus we do not take into account heterogeneity of land quality. In practice, at least some mixed farms are mixed precisely because some of the land is not suitable for crop production.

4 Empirical Application

4.1 Data description

Our empirical application focusses on a large sample of specialised and mixed farms in England and Wales. We obtain data from the Farm Business Survey (FBS) dataset covering the period 2007 – 2013. The FBS dataset provides farm-level information on economic and physical characteristics. It is unbalanced but statistically representative. Farms remain in the panel for a maximum of on average 5 – 7 years. To model the complex production processes on the farm in a detailed way, this paper exploits the rich characterisation of outputs and inputs of the FBS dataset. We distinguish 2 outputs, 12 variable inputs and 6 fixed factors. The outputs are crop production and livestock production. Joint non-reallocatable variable inputs are energy use, water use, hired labour and other

⁴In exceptional cases, this budget constraint may lead to $D_a(\mathbf{X}_{0,b}, \mathbf{Y}_{0,b}; \mathbf{g}_b | R) < D_a(\mathbf{X}_{0,b}, \mathbf{Y}_{0,b}; \mathbf{g}_b | NR)$ for $(a, b) \in \{t, t+1\} \times \{t, t+1\}$ and $a \neq b$ if the technically efficient allocations fall outside the budget constraint. One can solve this by using expenditures instead of (implicit) quantities, as prices are effectively equal to unity when using expenditures. However, using expenditures conflates technical and economic efficiency.

⁵This assumption can be weakened by assuming that reallocation leads to temporary reductions in production. We refer to [Oude Lansink and Stefanou \(2001\)](#), [Nemoto and Goto \(1999, 2003\)](#) and [Silva and Stefanou \(2003, 2007\)](#) for specific examples to model these adjustment costs.

inputs (costs on insurance, bank charges, professional fees, vehicle tax and other general farming costs). Crop-specific inputs are seed and young plants, fertilisers, crop protection and other variable crop costs. Livestock-specific inputs are bought feed and fodder, veterinary costs and medicine, and other livestock costs and the non-marketed crop output used as feed. Family labour and joint capital costs are joint non-reallocatable fixed factors. Aggregated crop-specific capital costs (permanent crops, debtors of crop subsidies, off-farm grain storage, crops, cultivations and stores) and livestock-specific capital costs (livestock and forage) are crop- and livestock-specific fixed factors, respectively. Crop land and livestock land are assumed to be fixed factors that are reallocatable among the outputs. This implies that total land use (and thus also farm size) is assumed to be fixed for a given year, but that the farmer can choose how much land to allocate to crop production and livestock production. Except for (hired and family) labour and land, which are measured in annual working hours and hectares, respectively, all inputs and outputs are measured in constant 2007 £. We compute implicit quantities of outputs and capital costs by calculating the ratio of value to the respective price index. We aggregate the monetary crop-specific, livestock-specific and joint variable inputs as implicit quantities by computing the ratio of their aggregated value to their corresponding aggregated Törnqvist price index. Price indices vary over the years but not over the farms, implying that differences in the composition or quality of inputs and outputs are reflected by differences in implicit quantity (Cox and Wohlgemant, 1986). The separate price indices are obtained from the Eurostat (2015) database.

Data Envelopment Analysis (DEA) is sensitive to different environmental conditions (*e.g.* weather conditions), outliers and measurement errors. We address these drawbacks as follows. First, we control for environmental differences by separately running the DEA models per region. East Midlands (EM), East of England (EE), South East (SE), North East (NE), North West (NW), Yorkshire & the Humber (YH), South West (SW), West Midlands (WM) and Wales (WA) are the considered regions. Second, we remove influential outliers using the approach developed by Banker and Chang (2006). We run DEA model (B2) for each observation by excluding the observation itself from the reference technology. Outliers are situated well outside the adjusted reference technology and appear 'super-efficient' (Banker and Chang, 2006) with a score substantially below zero. We only include the observations with a $D_t(\cdot|NR)$ between the 5 and 95 percentile.⁶ Since we explicitly account for heterogeneous technologies in our specification, we include specialised as well as mixed farms in our analysis. The eventual dataset contains

⁶Infeasibilities may appear when the considered observation has a peer with a projected negative output. Ray (2008) shows that these observations then have a score of lower than -0.5. We treat these observations as outliers.

12,738 observations for a period of seven years.

Table 1 shows the descriptive statistics of the variables used in the analysis.

Variables		Dimensions	Average	Std. Dev.
Crop-specific variable inputs	X_t^C	Constant 2007 £	47,731	144,414
Livestock-specific variable inputs	X_t^L	Constant 2007 £	15,160	28,083
Non-labour joint variable inputs	$Q_t^f, f \notin F$	Constant 2007 £	19,894	36,630
Hired labour		Ann. Working Hours	4,977	15,592
Family labour	$Q_t^f, f \in F$	Ann. Working Hours	2,631	969
Joint capital		£	1,086,669	1,505,083
Crop-specific capital	$X_t^{C,m}, m \notin C$	Constant 2007 £	85,416	250,381
Livestock-specific capital	$X_t^{L,m}, m \notin C$	Constant 2007 £	90,660	104,562
Crop land	$X_t^{C,m}, m \in C$	Hectares	131	334
Livestock land	$X_t^{L,m}, m \in C$	Hectares	224	286
Total crop output	$Y_t^C + Z_t^C$	Constant 2007 £	94,271	323,051
Crop output used as feed	Z_t^C	Constant 2007 £	2,449	7,781
Livestock output	Y_t^L	Constant 2007 £	165,403	376,680

Table 1: Descriptive statistics of variables

4.2 Static analysis: decomposing technical inefficiency

Table 2 presents the results of the static analysis of coordination inefficiency, CI , technical inefficiency when process-specific inputs over crops and livestock are optimally chosen, $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|R)$, and technical inefficiency when reallocation of process-specific inputs is not allowed, $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|NR)$.⁷ $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|NR)$ ranges from 0.044 (in NE) to 0.131 (in WA). Considering our specification of directional vectors, this means that farms in NE and WA could simultaneously expand their output levels and contract their input levels by on average 4.4% and 13.1%, respectively if their land use would remain fixed. These regions also provide the lowest (0.092) and highest (0.194) corresponding $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|R)$ if land use would be optimally reallocated. The wedge between $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|R)$ and $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|NR)$, CI , is on average small and ranges from 0.036 (in EM) to 0.063 (in WA and WM). Thus, several regions may reduce technical inefficiency by optimally diverting land use to livestock and crops.

This table also analyses the differences in CI , $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|R)$ and $D_t(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t|NR)$ among livestock farms (crop production covers 0-33% of total utilised land area), mixed farms (livestock production/crop production covers 33-66% of total utilised land area) and crop farms (livestock production covers 0-33% of total utilised land area).

⁷We only include the arithmetic averages to conserve space, but yearly results are available from the authors upon request.

Table 2 shows a clear pattern in differences in CI regarding farm types. CI in livestock farms is higher than CI in crop farms. CI in mixed farms is higher (lower) than CI in crop (livestock) farms. Specialisation in crop production thus leads to reduction in coordination inefficiency and better allocation of process-specific inputs. In what follows, we discuss the results that are significant at the 10% level using the Wilcoxon rank test (see Table A1 in the online Appendix). CI is significantly higher in livestock farms than in crop farms for all regions. Livestock farms have a significantly higher CI than do mixed farms in NW, SW and WA. In EM, CI is significantly higher in mixed farms than in livestock farms, but the difference is very small (0.006). In the majority of regions (EE, EM, NW, SE, SW and WM), CI is higher in mixed farms than in crop farms. In summary, these differences in CI are not only statistically significant, but also economically significant.

Turning to the non-reallocative directional distance function $D_t(\cdot|NR)$, no such pattern is present: in some regions (NW, SE, SW, WA, WM and YH) $D_t(\cdot|NR)$ of livestock farms is higher than for crop farms, while in others (EE, EM and NE) the opposite holds. This result is significant for the majority of the regions. Similar ambiguity holds for comparing mixed farms to specialised farms.

These inefficiencies are generally lower than those found in the efficiency literature on agriculture in the United Kingdom.⁸ However, it is difficult to compare our results with those in the literature. All previous results use radial measures focussing solely on input reductions or output expansions. In addition, all (but Hadley et al. (2013)) employ Stochastic Frontier Analysis where part of the inefficiency is explained as random noise. Finally, previous studies use a subsample of our sample by only focussing on one type of farms or using data spanning different periods.

Table 3 and Figure 3 analyse the land use changes that are associated with eliminating coordination inefficiency. If $CI = 0$, farms do not need to change land use (n.). If $CI > 0$, (B2) also allows us to compute the optimal land allocations. Farms should then either allocate more land to livestock (-) or crops (+). Interestingly, the required reallocation is considerably skewed towards more crop land use. For almost every region, there is a higher proportion of farms that would need to allocate more land to crops than to livestock. This holds for livestock farms, mixed farms as well as crop farms. This implies gains from specialisation (diversification towards mixed farming) for crop (livestock) farms. This confirms the above finding that livestock farms have more scope to reduce inefficiency by reallocating process-specific inputs.

These results hold only to a much lesser extent to WA, where the majority of

⁸Similar studies were conducted by Areal et al. (2012); Hadley (2006); Hadley et al. (2013); Karagiannis et al. (2002, 2004); Wilson et al. (2001).

Region		Total sample	Livestock	Mixed	Crops
EE	CI	0.037	0.073	0.057	0.027
	$D_t(\cdot R)$	0.139	0.145	0.164	0.138
	$D_t(\cdot NR)$	0.102	0.071	0.107	0.110
EM	CI	0.036	0.046	0.052	0.030
	$D_t(\cdot R)$	0.095	0.086	0.138	0.099
	$D_t(\cdot NR)$	0.059	0.040	0.085	0.070
NE	CI	0.048	0.049	0.050	0.044
	$D_t(\cdot R)$	0.092	0.093	0.114	0.089
	$D_t(\cdot NR)$	0.044	0.044	0.064	0.045
NW	CI	0.044	0.048	0.044	0.025
	$D_t(\cdot R)$	0.112	0.122	0.105	0.069
	$D_t(\cdot NR)$	0.069	0.074	0.061	0.044
SE	CI	0.042	0.063	0.050	0.028
	$D_t(\cdot R)$	0.112	0.137	0.136	0.095
	$D_t(\cdot NR)$	0.070	0.074	0.086	0.067
SW	CI	0.053	0.059	0.048	0.036
	$D_t(\cdot R)$	0.175	0.186	0.185	0.145
	$D_t(\cdot NR)$	0.122	0.127	0.137	0.109
WA	CI	0.063	0.064	0.038	0.034
	$D_t(\cdot R)$	0.194	0.196	0.087	0.069
	$D_t(\cdot NR)$	0.131	0.132	0.049	0.035
WM	CI	0.063	0.073	0.069	0.043
	$D_t(\cdot R)$	0.170	0.192	0.184	0.131
	$D_t(\cdot NR)$	0.108	0.119	0.115	0.087
YH	CI	0.048	0.053	0.039	0.043
	$D_t(\cdot R)$	0.100	0.105	0.117	0.093
	$D_t(\cdot NR)$	0.052	0.053	0.078	0.051

Table 2: Average static coordination inefficiency per region and level of specialisation

farms (63.6%) should not change their land allocation although the overwhelming majority of farms are livestock farms.

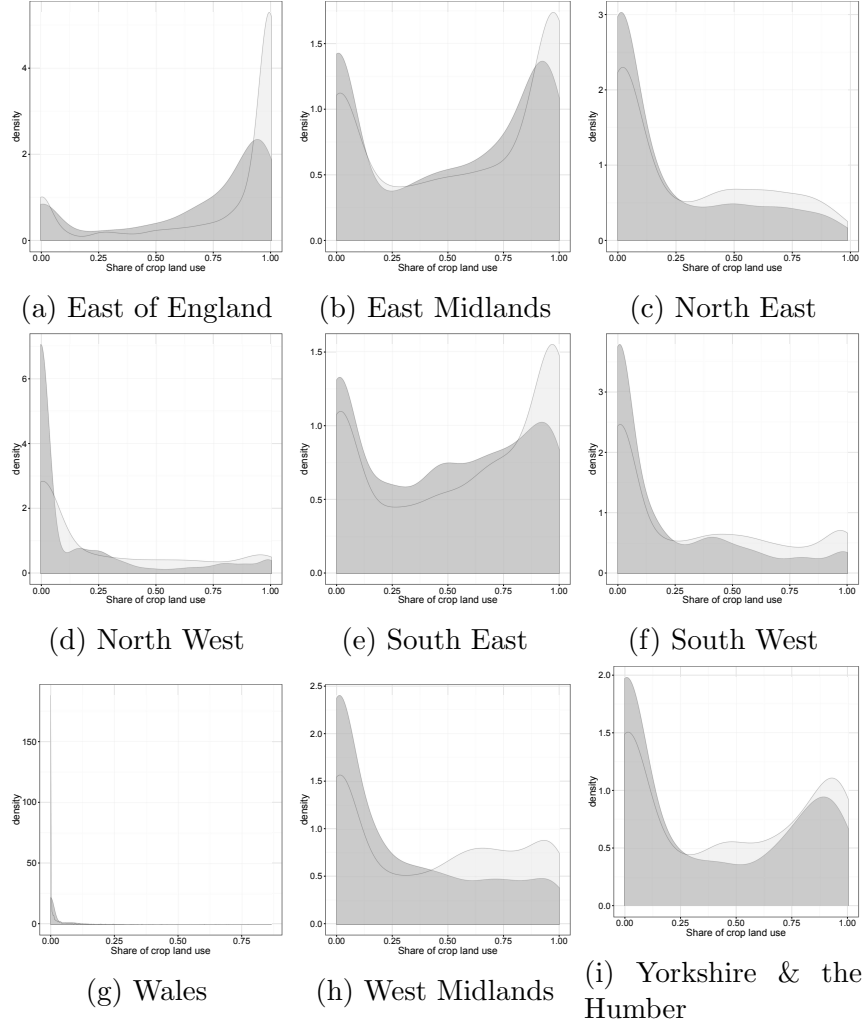


Figure 3: Distribution of optimal (□) and actual (■) land allocation in function of the proportion of land allocated to crops.

4.3 Dynamic analysis: decomposing Luenberger productivity growth

Table 4 presents the average coordination Luenberger productivity growth and its decomposition into coordination technical change ΔCT and coordination inefficiency change ΔCI for all regions. The chosen directional vectors ensure that

region	Livestock			Mixed: 50 – 66% Livestock			Mixed: 50 – 66% Crops			Crops		
	-	n.	+	-	n.	+	-	n.	+	-	n.	+
EE	0.022	0.078	0.099	0.014	0.000	0.040	0.015	0.004	0.066	0.118	0.080	0.465
EM	0.036	0.172	0.170	0.039	0.008	0.043	0.037	0.005	0.049	0.106	0.051	0.285
NE	0.132	0.279	0.308	0.023	0.002	0.055	0.028	0.010	0.036	0.051	0.011	0.065
NW	0.123	0.381	0.315	0.018	0.002	0.021	0.010	0.002	0.014	0.026	0.024	0.064
SE	0.068	0.135	0.192	0.049	0.002	0.061	0.041	0.001	0.079	0.093	0.082	0.199
SW	0.114	0.281	0.331	0.034	0.002	0.061	0.019	0.002	0.044	0.022	0.031	0.059
WA	0.169	0.636	0.182	0.005	0.001	0.003	0.002	0.000	0.000	0.000	0.000	0.001
WM	0.062	0.233	0.349	0.021	0.002	0.071	0.027	0.004	0.046	0.044	0.027	0.116
YH	0.072	0.227	0.251	0.040	0.005	0.029	0.025	0.005	0.026	0.115	0.032	0.174

Table 3: Share of farms that should allocate more land to livestock (-), crops (+) or which mix should remain unchanged (n.) (averaged over all years).

all contemporaneous DEA scores are feasible. Infeasibilities may arise for the components where the time period of the observation differs from the time period of the reference technology. No easy solutions exist to avoid infeasibilities. [Brieu and Kerstens \(2009\)](#) therefore recommend to simply report the number of infeasibilities (Table A3 in the online Appendix). The share of infeasibilities is very to moderately small, ranging from 4.67% to 23.67%.

Depending on the region, average $L_{t,t+1}(NR)$ per year ranges from -29.5% to 8.8% . Whereas annual average productivity declines in EM (-29.5%), NE (-3.3%), SE (-7.3%), SW (-1.1%) and YH (-4.1%), it increases in EE (4.9%), NW (8.8%), WA (0.8%) and WM (1.7%). The average coordination Luenberger productivity growth ranges from -9.7% to 15.9% , depending on the region. This is driven by ΔCT rather than ΔCI . The ability to reallocate process-specific inputs over time does not change substantially, whereas changes in the technology due to reallocation plays an important role.

In what follows, we only discuss the results that are statistically significant at the 10% level according to the Kolmogorov-Smirnov test reported in Table A2 in the online Appendix. Except for WM and YH, there are no significant differences in distribution of $L_{t,t+1}(NR)$ according to farm types. In contrast, the distributions of $CL_{t,t+1}$ differ according to farm types, although the sign of the statistical dominance is unclear in all regions except for NE.

Figure 4 shows the average coordination Luenberger productivity growth over time for each region. In every region, $CL_{t,t+1}$ is driven by ΔCT . Several large fluctuations occur for $CL_{t,t+1}$ and ΔCT between some years, which may be caused by weather conditions or by a few frontier farms that drive ΔCT .

Although the results show a clear pattern, we remain cautious as we have not taken into account heterogeneity of land quality. We cannot rule out the possibility that the results are partly an artifact of the data and the assumption of complete substitutability between crop land use and livestock land use. This may be an issue especially in regions with heterogeneous soils (*e.g.* NE and WA).

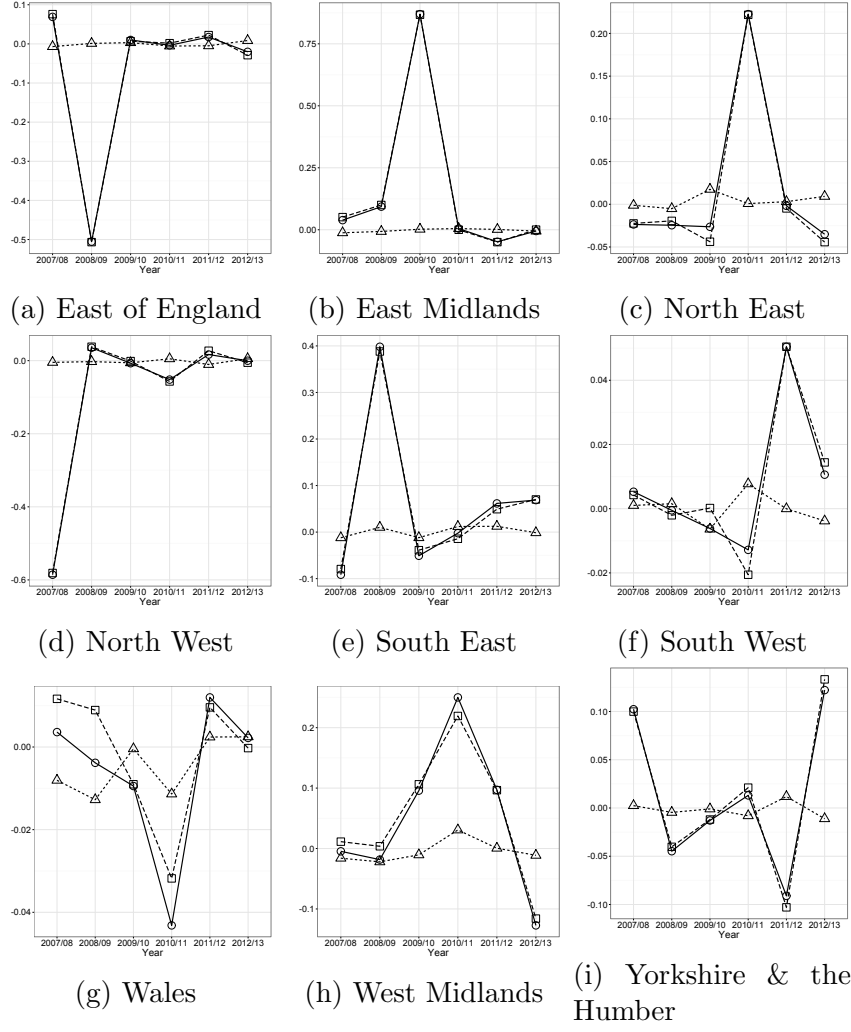


Figure 4: Decomposition of $CL_{t,t+1}$ ($\text{---}\bigcirc\text{---}$) in ΔCI ($\text{---}\triangle\text{---}$) and ΔCT ($\text{---}\square\text{---}$) per region.

	regions								
	EE	EM	NE	NW	SE	SW	WA	WM	YH
$L_{t,t+1}(NR)$	0.049	-0.295	-0.033	0.088	-0.073	-0.011	0.008	0.017	-0.041
$L_{t,t+1}(NR)$ for livestock farms	-0.009	-0.165	-0.035	0.107	-0.031	-0.017	0.007	0.015	0.044
$L_{t,t+1}(NR)$ for mixed farms	-0.025	-1.364	-0.004	-0.037	-0.191	-0.001	0.083	-0.017	-0.012
$L_{t,t+1}(NR)$ for crop farms	0.061	-0.374	-0.028	-0.015	-0.095	0.005	0.083	0.021	-0.135
$CL_{t,t+1}$	-0.072	0.159	0.019	-0.097	0.059	0.008	-0.006	0.051	0.014
$CL_{t,t+1}$ for livestock farms	-0.042	0.043	0.029	-0.112	0.027	0.014	-0.006	0.086	-0.048
$CL_{t,t+1}$ for mixed farms	-0.003	0.856	-0.030	-0.004	0.183	-0.005	-0.061	0.029	-0.015
$CL_{t,t+1}$ for crop farms	-0.079	0.228	-0.008	-0.013	0.077	-0.007	-0.061	-0.016	0.083
ΔCT	-0.072	0.161	0.015	-0.094	0.058	0.008	-0.002	0.056	0.016
ΔCT for livestock farms	-0.041	0.045	0.027	-0.111	0.026	0.015	-0.002	0.093	-0.045
ΔCT for mixed farms	0.002	0.865	-0.046	0.017	0.179	-0.010	-0.059	0.026	-0.014
ΔCT for crop farms	-0.078	0.231	-0.016	-0.004	0.075	-0.012	-0.059	-0.015	0.084
ΔCI	-0.001	-0.003	0.004	-0.002	0.001	0.000	-0.004	-0.005	-0.002
ΔCI for livestock farms	-0.001	-0.002	0.002	-0.001	0.001	-0.001	-0.004	-0.007	-0.003
ΔCI for mixed farms	-0.004	-0.009	0.016	-0.021	0.004	0.004	-0.001	0.003	-0.001
ΔCI for crop farms	-0.001	-0.003	0.009	-0.009	0.002	0.004	-0.001	-0.001	-0.000

Table 4: Average Luenberger productivity change and its components.

5 Conclusions

This paper develops a nonparametric measure of coordination Luenberger productivity growth where the subprocesses are explicitly modelled in the production technology. This indicator allows us to assess the change in the farmers' ability to allocate inputs over crop and livestock outputs over time. Focussing on a large panel of English and Welsh farms over the period 2007 – 2013, this paper demonstrates how better coordination of process-specific inputs may increase efficiency and productivity. We decompose coordination Luenberger productivity growth into coordination technical change and coordination inefficiency change. We compute the efficiency and productivity measures separately per region.

The static analysis shows a clear pattern: crop farms have a lower coordination inefficiency than livestock farms, *i.e.* they allocate their process-specific inputs more adequately. This result is statistically significant across all regions. Furthermore, coordination inefficiency in mixed farms is higher (lower) than coordination inefficiency in crop (livestock) farms. Coordination efficiency gains are associated with allocating more land use to crop production. In contrast, no such pattern exists considering the results for the non-reallocative directional distance function, which is now the standard way of measuring technical inefficiency. This demonstrates that richer modelling of subprocesses uncovers an additional source of inefficiency due to misallocation of resources.

According to the dynamic analysis, average non-reallocative Luenberger productivity growth per year ranges from -29.5% to 8.8% , with considerable differences across regions. The Kolmogorov-Smirnov test finds almost no significant distributional differences in farm types. We further find that average coordination Luenberger productivity growth ranges from -9.7% to 15.9% , depending on the

region. This is driven by coordination technical change rather than coordination inefficiency change. The ability to reallocate process-specific inputs over time does not change substantially, whereas changes in the technology due to reallocation plays an important role. The Kolmogorov-Smirnov test shows significant distributional differences in farm types, which contrasts the findings regarding the non-reallocative Luenberger productivity indicator. However, we find inconclusive evidence about which farm type stochastically dominates. Again, modelling subprocesses and allowing for reallocation reveal differences in optimally allocating resources over time. These differences are linked to heterogeneity in production technologies of different farm types.

Although researchers and policy makers identified an interest in stimulating mixed agriculture due to its environmental benefits, our results indicate that caution may be required. Since coordination efficiency gains are generally associated with more crop production for all farm types, one should stimulate mixed farming in livestock farms rather than crop farms. However, this does not necessarily imply that crop farms are more able to optimally allocate resources over time. Despite the clear patterns in the results, we remain prudent about the policy implications as the clear patterns may partly be an artifact of the data and the assumption of complete substitutability between crop land use and livestock land use.

We have several recommendations for future research. First, we recommend opening the black-box of efficiency and productivity by explicitly modelling the subprocesses. This can guide decision makers in coordinating the subprocesses to enhance efficiency and productivity. Second, this framework can be extended by including stochastic factors. Agricultural production is impacted by weather conditions, which cannot be influenced by the farms through choices of inputs and outputs. Efficiency is biased downwards (upwards) under bad (good) weather conditions. We have only partially controlled for this issue by running the nonparametric models per region. This problem can be dealt with in a more structural way by using Stochastic Frontier Analysis or making DEA conditional on environmental variables (de Witte and Kortelainen, 2013; Jeong et al., 2010). Finally, this framework can be augmented by taking into account intertemporal linkages. For instance, manure from livestock enterprises can be modelled as future inputs of crop production. Applied to the context of English and Welsh agriculture, this will be possible if more fertiliser surveys become available.

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A Additional Tables

		Region								
		EE	EM	NE	NW	SE	SW	WA	WM	YH
CI	Livestock - Mixed	0.227	0.028	0.920	0.057	0.676	0.016	0.001	0.752	0.346
	Livestock - Crops	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.000	0.003
	Crops - Mixed	0.000	0.000	0.228	0.008	0.000	0.000	0.692	0.000	0.278
$D_t(\cdot R)$	Livestock - Mixed	0.117	0.000	0.166	0.097	0.274	0.903	0.000	0.673	0.489
	Livestock - Crops	0.583	0.111	0.627	0.000	0.000	0.000	0.000	0.000	0.299
	Crops - Mixed	0.127	0.001	0.130	0.024	0.000	0.000	0.673	0.001	0.197
$D_t(\cdot NR)$	Livestock - Mixed	0.000	0.000	0.089	0.357	0.016	0.263	0.000	0.935	0.030
	Livestock - Crops	0.000	0.000	0.648	0.000	0.728	0.001	0.000	0.001	0.908
	Crops - Mixed	0.550	0.072	0.083	0.060	0.013	0.001	0.762	0.011	0.038

Table A1: P-values of Wilcoxon rank test.

		Region								
		EE	EM	NE	NW	SE	SW	WA	WM	YH
$L_{t,t+1}(NR)$	Livestock - Mixed	0.553	0.547	0.547	0.147	0.917	0.835	0.291	0.061	0.640
	Livestock - Crops	0.274	0.254	0.183	0.123	0.957	0.563	0.291	0.148	0.046
	Crops - Mixed	0.993	1.000	1.000	0.722	0.998	1.000	1.000	0.980	0.812
$CL_{t,t+1}$	Livestock - Mixed	0.179	0.410	0.338	0.784	0.073	0.334	0.037	0.102	0.515
	Livestock - Crops	0.000	0.000	0.846	0.090	0.000	0.001	0.037	0.002	0.017
	Crops - Mixed	0.001	0.003	0.995	0.330	0.014	0.091	1.000	0.366	0.799
ΔCT	Livestock - Mixed	0.149	0.878	0.503	0.338	0.202	0.080	0.049	0.501	0.626
	Livestock - Crops	0.000	0.000	0.603	0.005	0.000	0.000	0.049	0.002	0.008
	Crops - Mixed	0.002	0.001	0.885	0.083	0.027	0.107	1.000	0.564	0.202
ΔCI	Livestock - Mixed	0.154	0.437	0.313	0.548	0.985	0.082	0.229	0.648	0.743
	Livestock - Crops	0.000	0.001	0.587	0.020	0.001	0.000	0.229	0.001	0.068
	Crops - Mixed	0.059	0.002	0.894	0.271	0.016	0.067	1.000	0.496	0.962

Table A2: P-values of Kolmogorov-Smirnov test for non-reallocative Luenberger productivity growth, coordination Luenberger productivity growth, coordination technical change and coordination inefficiency change.

Region	$D_{t+1}(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t NR)$	$D_t(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1} NR)$	$D_{t+1}(\mathbf{X}_t, \mathbf{Y}_t; \mathbf{g}_t R)$	$D_t(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}; \mathbf{g}_{t+1} R)$
EE	14.06	13.75	8.97	8.61
EM	13.67	13.99	10.32	9.36
NE	21.63	16.79	15.75	12.26
NW	14.95	13.52	11.42	9.66
SE	16.92	11.70	12.10	8.09
SW	9.52	9.18	7.84	6.84
WA	6.92	9.49	4.67	7.05
WM	18.49	12.71	14.17	8.55
YH	23.67	17.11	18.35	12.62

Table A3: Share of infeasibilities over all years (in %).

B Linear Programmes

$$D_t(\mathbf{X}_{0,t}, \mathbf{Y}_{0,t}; \mathbf{g}_t = (\mathbf{g}_{x,t}, \mathbf{g}_{y,t}) | NR) =$$

$$\begin{aligned} & \max_{\substack{\beta, \\ \lambda_{k,t}, \gamma_{k,t} \geq 0}} \beta \\ \text{s.t. } & \sum_{k=1}^K \lambda_{k,t} X_{k,t}^{C,m} \leq X_{0,t}^{C,m} - \beta g_{x,t}^{C,m} & \forall m \notin C, \quad (\text{B1a}) \\ & \sum_{k=1}^K \lambda_{k,t} X_{k,t}^{C,m} \leq X_{0,t}^{C,m} & \forall m \in C, \quad (\text{B1b}) \\ & \sum_{k=1}^K \lambda_{k,t} Q_{k,t}^f \leq Q_{0,t}^f - \beta g_{Q,t}^f & \forall f \notin F, \quad (\text{B1c}) \\ & \sum_{k=1}^K \lambda_{k,t} Q_{k,t}^f \leq Q_{0,t}^f & \forall f \in F, \quad (\text{B1d}) \\ & \sum_{k=1}^K \lambda_{k,t} (\mathbf{Y}_{k,t}^C + \mathbf{Z}_{k,t}^C) \geq (\mathbf{Y}_{0,t}^C + \mathbf{Z}_{0,t}^C) + \beta \mathbf{g}_{y,t}^C, & (\text{B1e}) \\ & \sum_{k=1}^K \lambda_{k,t} = 1, & (\text{B1f}) \\ & \sum_{k=1}^K \gamma_{k,t} X_{k,t}^{L,m} \leq X_{0,t}^{L,m} - \beta g_{x,t}^{L,m} & \forall m \notin C, \forall m \notin H, \quad (\text{B1g}) \\ & \sum_{k=1}^K \gamma_{k,t} X_{k,t}^{L,m} \leq X_{0,t}^{L,m} & \forall m \in C, \quad (\text{B1h}) \\ & \sum_{k=1}^K \gamma_{k,t} (Z_{k,t}^{C,h} + X_{k,t}^{L,h}) \leq (Z_{0,t}^{C,h} + X_{0,t}^{L,h}) - \beta g_{x,t}^h & \forall h \in H, \quad (\text{B1i}) \\ & \sum_{k=1}^K \gamma_{k,t} Q_{k,t}^f \leq Q_{0,t}^f - \beta g_{Q,t}^f & \forall f \notin F, \quad (\text{B1j}) \\ & \sum_{k=1}^K \gamma_{k,t} Q_{k,t}^f \leq Q_{0,t}^f & \forall f \in F, \quad (\text{B1k}) \\ & \sum_{k=1}^K \gamma_{k,t} \mathbf{Y}_{k,t}^L \geq \mathbf{Y}_{0,t}^L + \beta \mathbf{g}_{y,t}^L, & (\text{B1l}) \\ & \sum_{k=1}^K \gamma_{k,t} = 1. & (\text{B1m}) \end{aligned}$$

$$D_t(\mathbf{X}_{0,t}, \mathbf{Y}_{0,t}; \mathbf{g}_t = (\mathbf{g}_{x,t}, \mathbf{g}_{y,t}) | R) = \max_{\substack{\beta, \\ \lambda_{k,t}, \gamma_{k,t} \geq 0, \\ \mathbf{x}_{0,t}^C, \mathbf{x}_{0,t}^L \geq 0}} \beta \quad (\text{B2a})$$

$$\text{s.t. (B1a) -- (B1m)} \quad (\text{B2b})$$

$$X_{0,t}^{C,m} + X_{0,t}^{L,m} = X_{0,t}^m \quad \forall m \in C \quad (\text{B2c})$$

$$\sum_{\forall i \in \{1, \dots, N_C\} \setminus C} p_{0,t}^{C,i} X_{0,t}^{C,i} + \sum_{\forall j \in \{1, \dots, N_L\} \setminus C} p_{0,t}^{L,j} X_{0,t}^{L,j} = PEXP_{0,t} \quad (\text{B2d})$$